On the Risk-Shifting Problem and Convertible Bonds

by

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The risk-shifting problem is one of several classic conflict of interest problems in corporate finance literature. Jensen and Meckling described the problem as one in which corporate management has the incentive to substitute a risky project for a less risky project and then showed that a risky bond issue would engender an agency cost that would be borne by stockholders. Subsequently, Green showed that a convertible bond issue could be used to eliminate the risk-shifting problem. In Green's model the corporation could invest in both the more and less risky projects. The purpose of this analysis is to consider the conflict of interest problem in the simpler setting in which the projects are mutually exclusive and to focus on both the conditions under which the risk-shifting problem exists and the structure of the convertible contracts that eliminate the problem.[1] In this setting of mutually exclusive projects, the convertible contract that eliminates the risk-shifting problem is not unique. The analysis generates a set of feasible contracts that solves the problem and shows that any convertible contract that allows for a greater increase in option value than the decrease in straight bond value is in this set.

Corporate management can encounter a conflict of interest problem in dealing with bondholders. Since the corporate manager represents the interests of stockholders and bondholders, there is a potential for conflict between the manager and bondholders, or equivalently, between the manager and the bondholders' trustee. This will be the case if it is possible for the manager to take actions that benefit one group and are detrimental to the other. If the bonds represent secured debt then there is no conflict. If not, then an agency problem may exist.

The agency relationship can be thought of as a contract between the principal (i.e., the bondholders' trustee)[2] and an agent (i.e., the corporate manager). The agent acts on behalf of

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[1]Green also assumed that one project's returns was not only a mean preserving spread of the others but also a value preserving spread. The analysis here does not employ the value preserving spread notion.

[2]The legal trustee for the bondholders may be treated as the single principal. It should be added that the trustee acts on behalf of the bondholders. The trustee's problem is the selection of bond covenants which limit the divergence of interests between corporate management and the bondholders. In general, the trustee may

the principal. The contract specifies the bounds on the actions that may be taken by the agent. If the contract covers all possible contingencies then there is no real delegation of authority and so no agency problem. If the contract is incomplete so that the agent has some discretion in the selection of actions then there is at least the potential for a conflict of interests. The conflict occurs because both the principal and the agent behave in accordance with their own self interests. The principal can limit the divergence of interests by providing provisions in the contract that give the agent the appropriate incentives to act in the principal's interest; in addition, the principal can monitor the activity of the agent. It is not usually possible, however, to specify the contract in such a way as to completely eliminate the conflict of interest problem.[3] Hence, it will usually be the case that there is a difference between the action taken by the agent and the action that is in the best interests of the principal. The agency cost is defined, by Jensen and Meckling, as the sum of the monitoring expenditures of the principal, the bonding expenditures of the agent, and the residual loss; this residual loss is the loss in the market value of the corporation.[4]

The agency problem considered here is encountered by the corporate management in selecting among mutually exclusive investment projects. Jensen and Smith noted that

> ". . . the value of the stockholders' equity rises and the value of the bondholders' claim is reduced when the firm substitutes high-risk for low-risk projects." [5]

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have a problem in selecting covenants which provide a solution to the conflict because of the different risk aversion measures of the bondholders. In the case considered here, however, the bondholders will unanimously support a covenant which provides management with the incentive to maximize the risk adjusted net present value of the corporation. It should also be noted that in general there may be an agency problem between the trustee and bondholders (i.e., between the agent and the principals). In the case considered here that problem does not arise because of the unanimity.

[3]In a complete market setting it is possible to take a normative approach by showing that a contract can be constructed which resolves a conflict of interest problem. Alternatively, it is possible to take a positive approach by showing that existing contractual forms can be used to resolve a conflict. The positive approach is taken here. The complete markets assumption simplifies the process of valuing the contracts.

[4]Jensen and Meckling also define the residual loss as the dollar equivalent of the loss in expected utility experienced by the principal. Although this notion of residual loss is measurable for a particular principal, this definition poses problems when a trustee represents many principals because the residual loss of any bondholder will depend on the bondholder's measure of risk aversion and on the proportion of the contract owned.

[5]See Michael Jensen and Clifford Smith, "Stockholder, Manager, and Creditor Interests: Applications of Agency Theory." Also see Richard Green, "Investment Incentives, Debt, and Warrants" and Clifford Smith and Jerold Warner, "On Financial Contracting: An Analysis of Bond Covenants," for similar statements.

Of course, bondholders are aware of this possibility, i.e., this attempt to shift risk, and so it is reflected in a lower value for the corporation's debt issue. The first purpose of this analysis is to demonstrate the conditions under which the risk-shifting problem exists and this is accomplished in section I. The second purpose is to characterize and construct the convertible bond packages that will reduce or eliminate the risk-shifting incentive. This is accomplished in section II.

I. Financial Markets and the Risk-Shifting Problem

Consider a competitive economy operating between the dates *now* and *then*. Let Ω denote the set of states of nature and let $\mathcal{B}$ denote the event space.[6] The set of states of nature is assumed to be finite. It follows that the event space is the set of all subsets of Ω. In a complete financial market system, it is possible to construct stock contracts which payoff one dolar in a particular state $\omega \in . \Omega$, and zero otherwise. Call these assets the basis stocks. Let the price of basis stock of type ω, be $p(\omega)$. The corporation that encounters the risk-shifting problem is treated separately. Call it corporation f and let p_f denote the share price of that corporation's stock issue. Let $\Pi_f(I_f, \omega)$ denote the firm's payoff where I_f is the capital value of the input and $\omega \in \Omega$ is the state of nature.

Let I denote the set of agents i in the economy. Let Ψ denote the agent's subjective probability distribution and let $\psi(\omega)$ denote the probability of state ω. Each agent has a utility function u_i: $D \to R$, $D \subset R^2$, which represents preferences for consumption now versus then. Consumption now is determinant but consumption then depends on the agent's decisions and the state of nature that occurs. The agents determine consumption now and then by purchasing\selling basis and corporate stock. The agent's expected utility is

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[6]The events of interest here are the bankruptcy and exercise events, i.e., B, $E \in \mathcal{B}$.

$$\sum_{\Omega} u_i(c_{i0}, c_{i1})\, \psi(\omega),$$

where the pair $c_i = (c_{i0}, c_{i1})$ represents consumption now and then, respectively. Each agent makes decisions now to maximize expected utility.

In the competitive and complete financial market system, the market value of corporation f is

$$V_f = \sum_{\Omega} p(\omega)\, \Pi_f(I_f, \omega).$$

Equivalently, the value of the incorporated firm is the risk adjusted present value of a portfolio of basis stock that has the same payoff structure as the corporation.

In order to demonstrate the agency problem, suppose the corporation is considering two mutually exclusive investment projects. Call them projects one and two. Let $I_1 = I_2$ denote the dollar cost of the two projects and let Π_1 and Π_2 denote the random project earnings.[7] Suppose project earnings are positive for all states. Suppose project two is riskier than project one, in the Rothschild-Stiglitz sense; in particular, let Π_2 be a mean-preserving spread of Π_1, as shown in figure one, i.e., $\Pi_2 = (1 + \delta)\, \Pi_1 - \delta\, E\Pi_1$, $\delta > 0$.[8] Then $\mu_2 = E\Pi_2 = E\Pi_1 = \mu_1$ and Π_2 has more weight in the tails of its distribution, as may be seen in figure one.

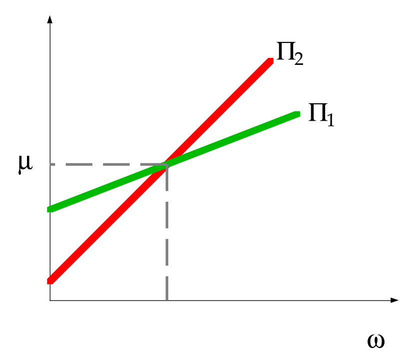

Figure 1

In this context, consider the statement by Jensen and Smith. Suppose bondholders believe that the firm will

[7]For notational simplicity, we drop the subscript indicating the corporation and replace it with a subscript indicating the project.

[8]The state space is still assumed to be finite but it is easier to see the mean-preserving spread when Π is drawn as a continuous function of ω.

select project one and value the bond issue accordingly. Next, suppose the firm switches to project two. Then the value of the bondholders' claim is reduced by the amount D_1 - D_2, as shown in figure two. Hence, there are circumstances under which it is not rational for the bondholders to believe any claim made by the firm that project one will be selected. Rational bondholders will protect their interests by considering what project will be selected by a management that acts in the best interests of stockholders.

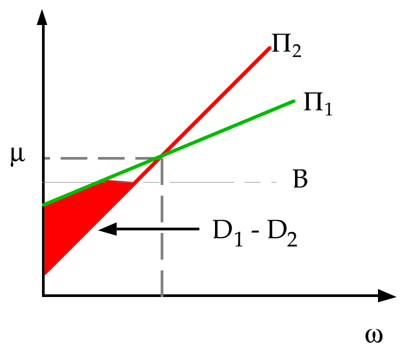

Figure 2

The first claim to be made here is that there exists a promised payment level B* such that $S_2^l > S_1^l$ for B > B* and $S_2^l < S_1^l$ for B < B*. This may be done simply by showing that in the unlevered case $S_1^u = V_1 > V_2 = S_2^u$ and in the sufficiently highly levered case $S_2^l > S_1^l$. Then the Intermediate Value Theorem yields the existence of a B*, since stock value is a continuous function of B.

Following the outlined procedure, it must be shown that if the corporation is unlevered and if project two is riskier than project one, then $S_1^u = V_1 > V_2 = S_2^u$. Let p denote the sum of the basis stock prices, or equivalently, the discount factor for a safe asset. Note that the difference in value is

$$V_1 - V_2 = \sum_\Omega p(\omega)\,\Pi_1(\omega) - \sum_\Omega p(\omega)\,\Pi_2(\omega = \sum_\Omega p(\omega)\,\delta\,(\mu_1 - \Pi_1(\omega)) = \delta\,\big(p\,\mu_1 - V_1\big) > 0,$$

since $V_1 < p\,\mu_1$, i.e., the risk adjusted present value of the project is less than the present value of a safe asset with the same expected payoff.[9] Therefore the value of the unlevered firm that selects project one is greater than the value of the unlevered firm that selects project two, i.e.,

[9]No matter how intuitive this may seem, it is just a claim and needs to be demonstrated. A proof of this claim is contained in the appendix.

$V_1 - V_2 = S_1^u - S_2^u > 0$. By continuity, it follows that $S_1^l - S_2^l > 0$ for B in a neighborhood of zero.

Second, it must be shown that in the highly levered case the stock market value of the riskier project is greater, i.e., $S_2^l > S_1^l$. To do this simply note that for B sufficiently large $\max\{0, \Pi_2 - B\} \geq \max\{0, \Pi_1 - B\}$ for all $\omega \in \Omega$ and strictly greater for some ω, as shown in figure three. Then it follows trivially that

$$S_2^l = \sum_\Omega p(\omega) \max\{0, \Pi_2(\omega) - B\}$$

$$> \sum_\Omega p(\omega) \max\{0, \Pi_1(\omega) - B\} = S_1^l$$

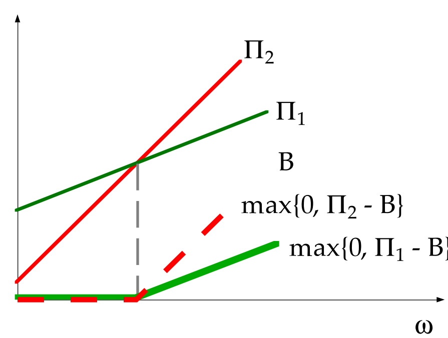

Figure 3

Then continuity completes the proof and a promised payment level B* exists such that $S_2^l > S_1^l$ for $B > B^*$ and $S_2^l < S_1^l$ for $0 < B < B^*$. Therefore, in the absence of any mechanism to avoid the agency problem, the corporation with a promised payment $B > B^*$ has an incentive to accept the riskier project. The agency cost of debt, in this case is

$$V_1 - V_2 = \sum_\Omega p(\omega) [\Pi_1(\omega) - \Pi_2(\omega)] > 0.$$

Actually, claiming that the corporation has an incentive to accept the riskier project is deceptive. Suppose the corporation must make a promised repayment of B_1 dollars in order to finance project one, i.e., $D_1(B_1) = I$, given that bondholders believe that project one will be selected. However, if $B_1 > B^*$ then the corporate management has a moral hazard problem since $S_2^l(B_1) > S_1^l(B_1)$. Rational bondholders understand this conflict of interest problem and will value the bond issue as if project two will be selected. Since $D_2(B_1) < I$, it follows that the promised repayment must be increased to $B_2 > B_1$ so that $D_2(B_2) = I$. The equity is appropriately priced as $S_2^l(B_2)$ but the agency cost is not directly reflected in this expression. It is possible to rewrite the stock market value $S_2^l(B_2)$ to reflect the agency cost. To do this, note that the promised payment B_2 must be selected so that

$$S_2^l(B_1) - S_2^l(B_2) = D_1(B_1) - D_2(B_1)$$ [10]

These differences in value are shown in the figure four. Note that $S_2^l(B_2)$ may be equivalently expressed as $S_2^l(B_2) = S_2^l(B_1) - [D_1(B_1) - D_2(B_1)]$. Also note that

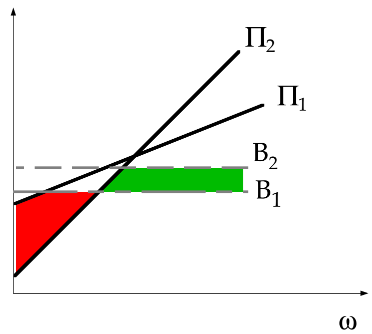

Figure 4

$$V_1 - V_2 = D_1(B_1) - D_2(B_1) + S_1^l(B_1) - S_2^l(B_1)$$ [11]

$$\Leftrightarrow S_2^l(B_1) = S_1^l(B_1) + [D_1(B_1) - D_2(B_1)] - [V_1 - V_2].$$

Then

$$S_2^l(B_2) = S_1^l(B_1) - [V_1 - V_2].$$

Hence, the stock market value of project two equals that of project one minus the agency cost of the bond issue. Observe that $S_2^l(B_2) < S_1^l(B_1)$. This makes it clear that it would be in the best interests of management to convince bondholders that project one will be selected. Because of the moral hazard problem, however, the straight bond contract cannot be used to convince them.

[10]This is equivalent to $S_2^l(B_1) = S_2^l(B_2) + D_1(B_1) - D_2(B_1)$ which simply says that if the bondholders believe that the firm will select project one and the firm selects project two then the stock market value of project two is its true value plus the devaluation of the bond issue.

[11]This follows by the MM 58 Theorem.

With a straight bond contract, the bondholders must expect the devaluation. What is more, at B_2, $S_2^l(B_2) > S_1^l(B_2)$ and the higher promised repayment locks management into the choice of project two.

II. Resolution of the Risk-Shifting Problem

Next, consider the value of convertible bonds. Suppose each bond is issued with a call option that gives its owner the right to exchange a bond for $\theta = \gamma\ (N + n)/B$ shares of stock, where n is the number of new shares that must be issued if the bonds are converted.[12] Note that a convertible bond is simply a bond with an attached call option. Also note that γ is the fraction of the firm's equity payoff that goes to bondholders in the event that the option is exercised. As has been shown, the payoff on the convertible bond issue is $\min\{\Pi, B\} + \max\{0, \gamma\Pi - B\}$, where B is the exercise value of the call option issue. Then it follows easily that the market value of the convertible bond issue is

$$D^c = \sum_{\Omega} p(\omega) \min\{\Pi(\omega), B\} + \sum_{\Omega} p(\omega) \max\{0, \gamma\Pi(\omega) - B\} = D + C.$$

Similarly, the value of the stock is

$$S^c = \sum_{\Omega} p(\omega) \max\{0, \Pi(\omega) - B\} - \sum_{\Omega} p(\omega) \max\{0, \gamma\Pi(\omega) - B\}$$

$$= \sum_{\Omega\backslash B} p(\omega) [\Pi(\omega) - B] - \sum_{E} p(\omega) [\gamma\ \Pi(\omega) - B] = S^l - C$$

i.e., the stock market value of the firm with convertible debt is equivalent to the stock market value of the firm with the simple debt contract minus the value of the option. In the second

[12]See MacMinn, "Lecture Notes on Valuing Financial Instruments."

expression for S^c, *B* is the bankruptcy event and *E* is the exercise event. Note that the bankruptcy event is $B = \{\omega \in \Omega \mid \Pi(\omega) < B\}$ and the exercise event is $E = \{\omega \in \Omega \mid \gamma\,\Pi(\omega) \geq B\}$, as shown in figure five. It may be noted that since the corporate value is not affected by the introduction of an option to convert, it follows that, *ceteris paribus*, the option reduces the stock market value.

Now, consider how management can construct a convertible issue that will convince the bondholders that project one will be selected. In fact, suppose management constructs the package to convince bondholders that project one will be selected and to maximize the stock market value of the corporation by eliminating the agency cost of the bond issue. Suppose management selects the provisions of the bond issue, i.e., (B, γ), so that $C_2 - C_1 > D_1 - D_2$; this is simply the condition that the difference in the value of the call options exceed the difference in the value of the straight bond issues. If bondholders believe that the firm will select project one and so value the bonds accordingly then by switching to project two the firm can capture the amount $D_1 - D_2$; equivalently, $D_1 - D_2$ represents the devaluation of the debt claim. $D_1 - D_2$ is represented by the lightly shaded area in figure five. Similarly, if project two is selected, given that bondholders believe project one will be selected, then $C_2 - C_1$ represents the increased value of the bondholders options. The net transfer of wealth to bondholders is $C_2 - C_1 - (D_1 - D_2)$. Note that

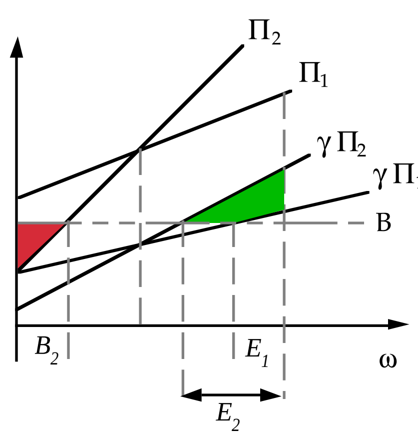

Figure 5

$$C_2 - C_1 > D_1 - D_2$$

$$\Leftrightarrow D_2 + C_2 > D_1 + C_1$$

$$\Leftrightarrow -D_2^c < -D_1^c$$

$$\Rightarrow V_2 - D_2^c < V_2 - D_1^c < V_1 - D_1^c$$

$$\Leftrightarrow S_2^c < S_1^c$$

Hence, any feasible convertible contract that satisfies the condition $C_2 - C_1 > D_1 - D_2$ will convince bondholders that firm management will select project one. Figure five represents the tradeoff between bond values and option values.

Having shown that it is possible to convince bondholders that project one will be selected, it remains to be shown that management has the incentive to select that project. This, however, is also a simple matter. Since the manager is an initial shareholder, all that must be shown is that a convertible contract (B, γ) exists such that the following three conditions are satisfied: (a) $C_2 - C_1 > D_1 - D_2$; (b) $D_1^c = I$; (c) $S_1^c(B, \gamma) > S_2^l(B_2)$. Conditions (a) and (b) ensure the bondholders believe that project one will be selected and that the bond issue covers the investment expenditure. Condition (c) shows that the manager and stockholders prefer project one. To establish condition (c) simply note that

$$S_1^c(B, \gamma) = V_1 - D_1^c(B, \gamma) = V_1 - I > V_2 - I = V_2 - D_2(B_2) = S_2^l(B_2)$$

The second equality follows by condition (b). Thus, managers and stockholders prefer this convertible bond contract. It may also be noted that this convertible contract preserves the stockholders' value since $S_1^c(B, \gamma) - S_2^l(B_2) = V_1 - V_2 > 0$, i.e., the contract completely eliminates the agency cost of debt.

Now, consider the set of convertible bond contracts that are feasible, finance project one, and solve the risk shifting problem. Since the pair (B, γ) completely specifies the contract, the feasible contract set may be written as $H = [0, \bar{B}] \times [0, 1]$, where $\bar{B}$ is the smallest promised payment that satisfies $D_1(\bar{B}) = V_1$. Then the convertible bond contract is a function D^c: H → R, such that

$$D^c(B, \gamma) = \sum_B p(\omega)\, \Pi(\omega) + \sum_{\Omega \setminus B} p(\omega)\, B + \sum_E p(\omega)\, [\gamma\, \Pi(\omega) - B]$$

$$= \sum_B p(\omega)\, \Pi(\omega) + \sum_D p(\omega)\, B + \sum_E p(\omega)\, \gamma\, \Pi(\omega),$$

where $D = \Omega \setminus (B \cup E)$, i.e., the event that the firm is not bankrupt and the bondholders do not exercise their conversion option. Let the function H : H → R be defined as

$$H(B, \gamma) = D^c(B, \gamma) - I.$$

Note that

$$D_1H = \sum_D p(\omega) > 0 \text{ if } D \neq \phi$$

and

$$D_2H = \sum_E p(\omega)\, \Pi(\omega) \text{ if } E \neq \phi.$$

Hence, for the contracts (B, γ) such that $E \neq \phi$, it follows by the Implicit Function Theorem that there exists a function h such that H(B, h(B)) = 0 and

$$h'(B) = -\frac{D_1H}{D_2H} = -\frac{\sum_D p(\omega)}{\sum_E p(\omega)\, \Pi(\omega)} \leq 0$$

where it exists. Since both partial derivatives are non-negative, h is a non-increasing function. In order to determine the properties of the relation $\gamma = h(B)$, let $\Omega = \{\omega_1, \omega_2, \omega_3\}$ and consider the following partitions of the convertible contract set H. Let

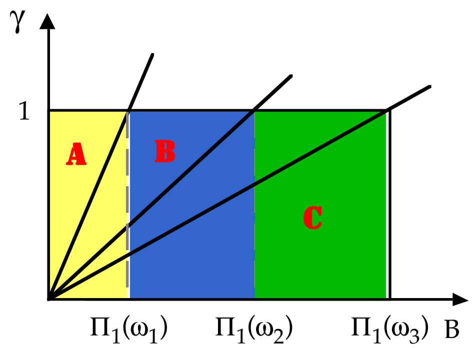

Figure 6

$$\mathbf{A} = \{(B, \gamma) \in H \mid B < \Pi_1(\omega_1), \gamma \in [0, 1]\}$$

$$\mathbf{B} = \{(B, \gamma) \in H \mid \Pi_1(\omega_1) \leq B < \Pi_1(\omega_2), \gamma \in [0, 1]\}$$

$$\mathbf{C} = \{(B, \gamma) \in H \mid \Pi_1(\omega_2) \leq B, \gamma \in [0, 1]\}.$$

Note that the bankruptcy event is empty for contracts in **A**, while it is $\{\omega_1\}$ and $\{\omega_1, \omega_2\}$ for contracts in **B** and **C**, respectively. These subsets of the contract set H are shown in figure six. Similarly, let

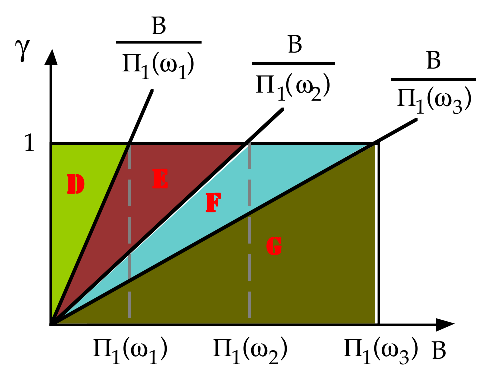

Figure 7

$$\mathbf{D} = \{(B, \gamma) \in H \mid \gamma > \frac{B}{\Pi_1(\omega_1)}, \gamma \in [0, 1]\}$$

$$\mathbf{E} = \{(B, \gamma) \in H \mid \frac{B}{\Pi_1(\omega_1)} \geq \gamma > \frac{B}{\Pi_1(\omega_2)}, \gamma \in [0, 1]\}$$

$$\mathbf{F} = \{(B, \gamma) \in H \mid \frac{B}{\Pi_1(\omega_2)} \geq \gamma > \frac{B}{\Pi_1(\omega_3)}, \gamma \in [0, 1]\}$$

$$\mathbf{G} = \{(B, \gamma) \in H \mid \frac{B}{\Pi_1(\omega_3)} \geq \gamma, \gamma \in [0, 1]\}.$$

Note that contracts in **D, E, F,** and **G** yield exercise events Ω, $\{\omega_2, \omega_3\}$, $\{\omega_3\}$, and ϕ, respectively. These subsets of H are shown in figure seven. As noted, there is a correspondence between subsets of the feasible contract set H and the events in $\mathcal{B}$. For example, contracts in **A** ∩ **D** yield the events $B = \phi = D$ and $E = \Omega = \{\omega_1, \omega_2, \omega_3\}$. The following table specifies the correspondence between the subsets of H and the events in $\mathcal{B}$.

	EVENTS		
SUBSETS	*B*	*D*	*E*
A ∩ **D**	ϕ	ϕ	Ω
A ∩ **E**	ϕ	$\{\omega_1\}$	$\{\omega_2, \omega_3\}$
A ∩ **F**	ϕ	$\{\omega_1, \omega_2\}$	$\{\omega_3\}$
A ∩ **G**	ϕ	Ω	ϕ
B ∩ **E**	$\{\omega_1\}$	ϕ	$\{\omega_2, \omega_3\}$
B ∩ **F**	$\{\omega_1\}$	$\{\omega_2\}$	$\{\omega_3\}$
B ∩ **G**	$\{\omega_1\}$	$\{\omega_1, \omega_2\}$	ϕ
C ∩ **F**	$\{\omega_1, \omega_2\}$	ϕ	$\{\omega_3\}$
C ∩ **G**	$\{\omega_1, \omega_2\}$	$\{\omega_3\}$	ϕ

Recall that h is not defined for $(B, \gamma) \in \mathbf{G}_1$.[13] Let $\hat{B}$ be the promised repayment on the zero coupon bonds necessary to raise the I dollars given no conversion feature and let $\hat{B} \in (\Pi_1(\omega_1), \Pi_1(\omega_2))$. Then the path of the function h will either be through $\mathbf{A}_1 \cap \mathbf{F}_1$ or $\mathbf{B}_1 \cap \mathbf{E}_1$. The path will depend on the risk adjusted present value of the project.[14] For the path through $\mathbf{B}_1 \cap \mathbf{E}_1$, it follows that

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[13]This case is considered in the subsequent analysis.

[14]Note that $(B, \gamma) = (0, \hat{\gamma})$ is a convertible bond contract which is equivalent to a straight stock issue. The stock value of the new issue is $S_1^n = \hat{\gamma}\, S_1$, where S_1 is the total stock value and $\hat{\gamma}$ is the fraction of the firm's equity issued to new equityholders. Then $\hat{\gamma}$ must be selected so that $S_1^n = I$, or equivalently,

$$\hat{\gamma} = \frac{I}{S_1}.$$

Also, note that increasing the corporate payoff increases S_1 and, other things being equal, decreases $\hat{\gamma}$.

$$h'(B) = \begin{cases} 0 & (B,\gamma) \in \mathbf{A}_1 \cap \mathbf{D}_1 \\ -\dfrac{p(\omega_1)}{p(\omega_2)\,\Pi_1(\omega_2) + p(\omega_3)\,\Pi_1(\omega_3)} & (B,\gamma) \in \mathbf{A}_1 \cap \mathbf{E}_1 \\ 0 & (B,\gamma) \in \mathbf{B}_1 \cap \mathbf{E}_1 \\ -\dfrac{p(\omega_2)}{p(\omega_3)\,\Pi_1(\omega_3)} & (B,\gamma) \in \mathbf{B}_1 \cap \mathbf{F}_1 \end{cases}$$

This contract line is shown in figure eight. Note that it is piece wise linear, i.e., h″ = 0 where it exists. For contracts in $\mathbf{A}_1 \cap \mathbf{D}_1$, investors know that call option will be exercised and so increasing the promised payment on the bonds will not add value, i.e., h′ = 0. For contracts in $(\mathbf{A}_1 \cap \mathbf{F}_1) \cup (\mathbf{B}_1 \cap \mathbf{F}_1)$, increasing the promised payment on the bonds will, other things being equal, increase the value of the straight debt component because there is one state in which the bonds will not be converted into stock and the bondholders will receive the increased payment; it follows that an increased payment must be associated with a decreased conversion proportion in order to maintain the same bond value. For contracts in $\mathbf{B}_1 \cap \mathbf{E}_1$, an increased promised payment does not add value because there is no state in which the increased payment would be received. For contracts in $\mathbf{B}_1 \cap \mathbf{G}_1$, increasing the conversion proportion will not add value because there is no state in which the bonds will be called.[15]

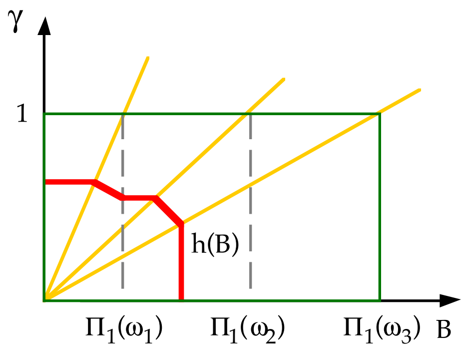

Figure 8

[15]Note that the Implicit Function Theorem does yield a function relationship between B and γ, since $D_1H > 0$. It follows that there exists a function $g : [0, 1] \rightarrow G_1$ such that $H(g(\gamma), \gamma) = 0$ and

$$g'(\gamma) = -\frac{D_2H}{D_1H} = -\frac{\Sigma_{E_1}\, p(\omega)\,\Pi_1(\omega)}{\Sigma_{D_1}\, p(\omega)} = 0$$

Next, consider the set of contracts in H such that value is independent of project selection, i.e., $D_1^c(B, \gamma) = D_2^c(B, \gamma)$. Recall that

$$D_j^c(B, \gamma) = \sum_{B_j} p(\omega)\, \Pi_j(\omega) + \sum_{D_j} p(\omega)\, B + \sum_{E_j} p(\omega)\, \gamma\, \Pi_j(\omega),$$

for j = 1, 2. Let the function G: $H \rightarrow R$ be defined as $G(B, \gamma) = D_2^c(B, \gamma) - D_1^c(B, \gamma)$. Then

$$D_1G = \sum_{D_2} p(\omega) - \sum_{D_1} p(\omega)$$

and

$$D_2G = \sum_{E_2} p(\omega)\, \Pi_2(\omega) - \sum_{E_1} p(\omega)\, \Pi_1(\omega).$$

For (B, γ) such that $E_1 \cup E_2 \neq \phi$, it follows by the Implicit Function Theorem that there exists a function g such that $G(B, g(B)) = 0$ and

$$g' = -\frac{D_1G}{D_2G} = \frac{\sum_{D_1} p(\omega) - \sum_{D_2} p(\omega)}{\sum_{E_2} p(\omega)\, \Pi_2(\omega) - \sum_{E_1} p(\omega)\, \Pi_1(\omega)}$$

where it exists. Then, using the table of correspondences, it follows that

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since $E_1 = \phi$ for all contracts in $\mathbf{G}_1$.

$$
g'(B)=\begin{cases}
\dfrac{1}{\Pi_2(\omega_3)} & (B,\gamma)\in(\mathbf{A_1}\cap\mathbf{G_1})\cap(\mathbf{A_2}\cap\mathbf{F_2})\\
\dfrac{p(\omega_1)+p(\omega_3)}{p(\omega_3)\,\Pi_2(\omega_3)} & (B,\gamma)\in(\mathbf{A_1}\cap\mathbf{G_1})\cap(\mathbf{B_2}\cap\mathbf{F_2})\\
\dfrac{1}{\Pi_2(\omega_3)} & (B,\gamma)\in(\mathbf{B_1}\cap\mathbf{G_1})\cap(\mathbf{B_2}\cap\mathbf{F_2})\\
\dfrac{1}{\Pi_2(\omega_3)} & (B,\gamma)\in(\mathbf{C_1}\cap\mathbf{G_1})\cap(\mathbf{C_2}\cap\mathbf{F_2})
\end{cases}
$$

The function g is shown in figure nine. For simplicity, it is assumed that $\mu = \Pi_1(\omega_2) = \Pi_2(\omega_2)$. As expected, g is an increasing function. Notice that all contracts (B, γ) above g yield $D_2^c(B, \gamma) > D_1^c(B, \gamma)$. Hence, it follows that all contracts on h and above g solve the risk shifting problem. Equivalently, the set of convertible contracts that solves the risk-shifting problem is the subset **K** of H such that h(B) ≥ g(B). It should be noted that the contract set **K** includes the equity contract (0, $\hat{\gamma}$) as a special case but it also includes risky bond instruments. In the absence of other conflict of interest problems, the manager and stockholders are indifferent between any pair of contracts in **K**. What is more important, however, all contracts in **K** make management and stockholders better off without making the bondholders any worse off and this yields the result that the capital structure of the firm is not entirely a matter of irrelevance!

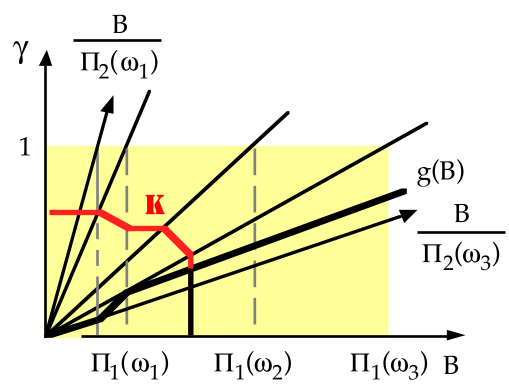

Figure 9

References

Arrow, Kenneth J. "The Role of Securities in the Optimal Allocation of Risk Bearing." Review of Economic Studies (1963):

DeAngelo, Harry. "Competition and Unanimity." American Economic Review 71.1 (1981): 18-27.

Fisher, Irving. The Theory of Interest. New York: MacMillan, 1930.

Garman, Mark. "A Synthesis of the Pure Theory of Arbitrage." Working Paper No. 98, Institute of Business and Economic Research. University of California at Berkeley, 1979.

Green, Richard C. "Investment Incentives, Debt and Warrants." Journal of Financial Economics 13 (1984): 115-36.

Jensen, M., and W. Meckling. "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." Journal of Financial Economics 3 (1976): 305-60.

Jensen, Michael, and Clifford Smith. "Stockholder, Manager, and Creditor Interests: Applications of Agency Theory." Recent Advances in Corporate Finance. Ed. Edward Altman and Marti Subrahmanyam. Richard D. Irwin, 1985.

MacMinn, Richard D., "Lecture Notes on Valuing Financial Instruments," University of Texas, March 1989.

Modigliani, Franco, and Merton H. Miller. "The Cost of Capital, Corporation Finance and the Theory of Investment." American Economic Review (1958):

Rothschild, Michael, and Joseph E. Stiglitz. "Increasing Risk: I. A Definition." Journal of Economic Theory 2 (1970): 225-43.

Smith, Clifford, and Jerold Warner. "On Financial Contracting: An Analysis of Bond Covenants." Journal of Financial Economics 7 (1979): 117-61.

Varian, Hal R. "Divergence of Opinion in Complete Markets." The Journal of Finance XL (1985): 309-17.

Appendix

Claim 1: If the quotient kernel $k(\omega) = p(\omega)/\psi(\omega)$ is decreasing in ω, then $p\,\mu \geq V$.[16]

Proof.

$$p\,\mu - V = p \sum_{\Omega} \Pi(\omega)\, \psi(\omega) - \sum_{\Omega} p(\omega)\, \Pi(\omega)$$

$$= \sum_{\Omega} \big(p\, \psi(\omega) - p(\omega)\big)\, \Pi(\omega).$$

Note that $p\, \psi(\omega)$ is greater than $p(\omega)$ for some ω and less than $p(\omega)$ for other ω. In addition, note that

$$\sum_{\Omega} \big(p\, \psi(\omega) - p(\omega)\big) = 0.$$

It follows trivially that if $\Pi(\omega)$ is a constant then $p\,\mu = V$, as expected. Also note that

$$p\,\mu - V = \sum_{\Omega} \big(p\, \psi(\omega) - p(\omega)\big)\, \Pi(\omega)$$

$$= \sum_{\Omega} \left(p - \frac{p(\omega)}{\psi(\omega)}\right) \Pi(\omega)\, \psi(\omega)$$

$$= p\,\mu - \sum_{\Omega} k(\omega)\, \Pi(\omega)\, \psi(\omega)$$

[16] In "A Synthesis of the Pure Theory of Arbitrage," Garman develops the quotient kernel and shows that it can be used to classify models.

$$= p\,\mu - Ek\,E\Pi - Cov(k, \Pi)$$

$$= p\,\mu - Ek\,\mu - Cov(k, \Pi)$$

$$= - Cov(k, \Pi)$$[17]

$$> 0,$$

since k is decreasing and Π is increasing in ω. Q.E.D.

Claim 2: k is a decreasing function.

Proof. The first order conditions for a maximum are

$$\textstyle\sum_{\Omega} D_1 u\, \psi(\omega) - \lambda = 0$$

$$D_2 u\, \psi(\omega) - \lambda\, p(\omega) = 0, \text{ for all } \omega \in \Omega$$

These first order conditions imply that

$$\frac{D_2 u}{\lambda} = \frac{p(\omega)}{\psi(\omega)} \equiv k(\omega).$$

Let the function F be defined as

$$F(c_1, k) \equiv \frac{D_2 u}{\lambda} - k.$$

[17]This equality follows because

$$Ek = \textstyle\sum_{\Omega} k(\omega)\ \psi(\omega) = \sum_{\Omega} p(\omega) = p.$$

By the Implicit Function Theorem, there exists a differentiable function f such that $F(f(k), k) = 0$ and

$$f' = -\frac{D_2F}{D_1F} = -\frac{-1}{\frac{D_{22}u}{\lambda}} = \frac{1}{\frac{D_{22}u}{\lambda}} < 0.$$

This holds for each investor $i \in I$. Now, let

$$c_1(\omega) = \sum_I c_{i1}(\omega) = \sum_I f_i(k(\omega)) = f(k(\omega)).$$

The RHS is a decreasing function since $f_i' < 0$ for all $i \in I$. This implies that there exists a function $h = f^{-1}$, or equivalently, that $k = h(c_1)$. Since $\Pi(\omega)$ increases with ω for all corporations, it follows that in equilibrium c_1 must increase in ω. Actually, all that seems necessary is that the sum of the corporate payoffs be increasing in ω. Then, since

$$h' = \frac{1}{f'} < 0,$$

it follows that k is a decreasing function of ω. Q.E.D.[18]

[18]See Hal Varian's "Divergence of Opinion in Complete Markets," for a similar conclusion.