On Rothschild and Stiglitz

Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information

by

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Consider an individual who will have an income w in the event of no accident and $w - d$ if an accident occurs. The individual can insure himself against the accident by paying a premium of d_1 ,in return for which he is paid the net amount δ_2 if an accident occurs, i.e. δ_2 is the gross benefit minus the premium. Without insurance the individual's income in the two states of nature is $(w, w - d)$ while with insurance it is $(w + \delta_2, w - d + \delta_2)$. Note that the premium is treated as a negative dollar amount while the net benefit is a positive dollar amount, i.e., $\delta_1 < 0$ and $\delta_2 > 0$. This simply says that the insurance contract is viewed from the perspective of the insured rather than the insurer. The vector $\delta = (\delta_1 + \delta_2)$ completely describes the insurance contract.

The Demand for Insurance Contracts

The individual demands a contract to alter his income across states of nature. Let $y = (y_1 + y_2)$ where y_i is income in state j. Expected utility is

$$V(p, y) = (1 - p)\, u(y_1) + p\, u(y_2)$$

where u is the utility function and p is the probability of an accident. Alternatively, the value of a contract δ may be expressed as $v(p,\delta) = v(p, w + \delta_1, w - d + \delta_2)$. This formulation allows us to work in contract space rather than income space. The individual selects the contract which maximizes $V(p, y)$ or $v(p,\delta)$ subject to the appropriate constraint. Obviously the individual only purchases insurance if there exists a δ such that $v(p,\delta) > v(p,0)$.

The Supply of Insurance Contracts

Suppose insurance companies are risk neutral. Then a contract □ sold to an individual with an accident probability p is worth $\pi(p,\delta) = -(1 - p)\delta_1 - p\delta_2 = -\delta_1 - p(\delta_2 - \delta_1)$. Assume free entry into the market. This ensures $\pi(p,\delta) = 0$ in equilibrium.

Information

Assume that consumers know their accident probabilities but insurance companies do not. Also suppose that consumers are identical in all respects except their propensity to have accidents.

The Nash Equilibrium Concept

Suppose consumers can only purchase one contract, i.e. firms are price and quantity setters. Then equilibrium in a competitive insurance market is a set of contracts such that when customers maximize expected utility

(i) no contract in the equilibrium set makes negative expected profits

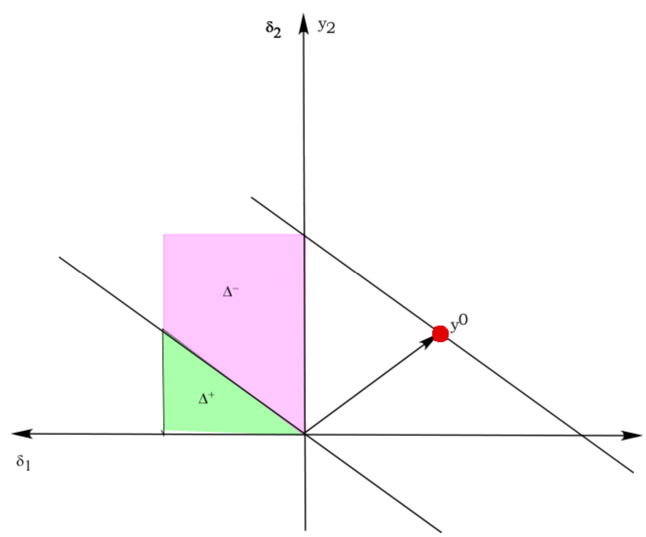
(ii) there is no contract outside the equilibrium that could make a non-negative profit if it was offered.

The Equilibrium with Identical Customers

In this case insurance companies have perfect information. Free entry ensures $-(1-p)\delta_1 - p\delta_2 = 0$ and this equation represents the set of contracts which break even. Let $\Delta = \{\delta | -(1-p)\delta_1 - p\delta_2 = 0\}$ and $y^0 = (w, w-d)$. Then $Y = y^0 + \Delta$ is the set of corresponding incomes in the two states of nature, i.e.,

$$Y = \{y = (y_1, y_2) \mid (1-p)y_1 + p\,y_2 = (1-p)w + p(w-d)\}$$

The set Y is called the fair odds line. The contract and income sets are represented in the following figure. Let Δ^+ and Δ^- denote the contract sets which yield positive and negative profits respectively, i.e., $\Delta^+ = \{\delta \mid \pi(p,\delta) > 0\}$ and $\Delta^- = \{\delta \mid \pi(p,\delta) < 0\}$. These contract sets are also shown in the following figure.



The policy δ that maximizes $v(p,\delta)$ and breaks even, i.e. it is the solution of the problem

$$\begin{gathered}\max\text{imize } v(p, \delta) \\ \text{subject to } \delta \in \Delta\end{gathered}$$

Or equivalently

$$\begin{gathered}\max\text{imize } V(p, y) \\ \text{subject to } y \in Y\end{gathered}$$

Note that any contract preferred to the optimal δ will yield an expected loss. The problems may be equivalently expressed using the Lagrange function L, i.e.,

$$\max\text{imize} \quad L(\delta, \lambda) = v(p, \delta) + \lambda\left((1-p)\delta_1 + p\delta_2\right) \tag{1}$$

The first order conditions are

$$\frac{\partial L}{\partial \delta_1} = \frac{\partial v}{\partial \delta_1} + \lambda(1-p) = 0 \tag{2}$$

$$\frac{\partial L}{\partial \delta_2} = \frac{\partial v}{\partial \delta_2} + \lambda p = 0 \tag{3}$$

and

$$\frac{\partial L}{\partial \lambda} = (1-p)\delta_1 + p\delta_2 = 0 \tag{4}$$

It follows that the condition for an optimal insurance contract is

$$\frac{\frac{\partial v}{\partial \delta_1}}{\frac{\partial v}{\partial \delta_2}} = \frac{(1-p)}{p} \tag{5}$$

where the left hand side (LHS) is the marginal rate of substitution for the expected utility function and the right hand side (RHS) is the absolute value of the slope of the constraint. Observe that

$$\frac{\partial v}{\partial \delta_1} = \frac{\partial}{\partial \delta_1}\left((1-p)\,u(w+\delta_1) + p\,u(w-d+\delta_2)\right) = (1-p)\,u'(w+\delta_1)$$

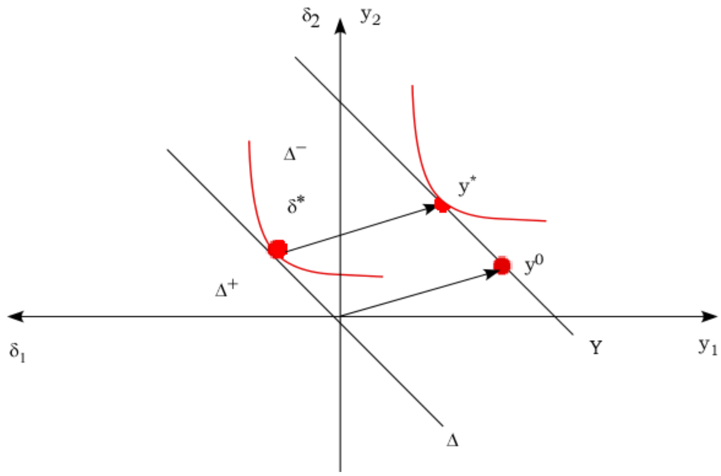
and

$$\frac{\partial v}{\partial \delta_2} = \frac{\partial}{\partial \delta_2}\left((1-p)\,u(w+\delta_1) + p\,u()\right) = p\,u'(w-d+\delta_2)$$

and so the condition for an optimal insurance policy can be equivalently expressed as

$$\frac{(1-p)u'(w+\delta_1)}{p\,u'(w-d+\delta_2)} = \frac{(1-p)}{p} \tag{6}$$

This condition holds if and only if $w+\delta_1 = w - d + \delta_2$, or equivalently, $d = \delta_2 - \delta_1$. Since customers are risk averse, $y^* = y^0 + \delta^*$ is located at the intersection of the 45 degree line and the fair odds line. In equilibrium each customer buys complete insurance, i.e., $d = \delta_2^* - \delta_1^*$.



The Asymmetric Information Case

Suppose the market consists of two kinds of customers: low risk individuals with accident probability p^L, and high-risk individuals with accident probability $p^H > p^L$ Let λ be the fraction of high-risk customers. Then the average accident probability is $\bar{p} = \lambda p^H + (1-\lambda)p^L$. This market can have either a pooling equilibrium in which both groups buy the same contract or a separating equilibrium in which different types purchase different contracts.

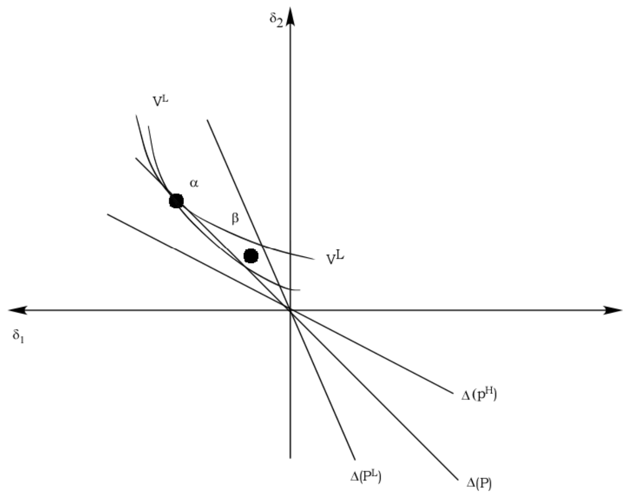
Rothschild and Stiglitz claim that a pooling contract cannot be an equilibrium contract. This notion is captured in the following proposition.

Pooling Proposition: Given asymmetric information, risk neutral insurers, and risk averse customers, a pooling contract cannot exist as an equilibrium contract.

Sketch of Proof. Let α be a pooling contract in $\Delta(\bar{p})$. It follows that $\pi(\bar{p},\alpha) = 0$. At the contract α, observe that

$$mrs^L \equiv \frac{\frac{\partial v^L}{\partial \delta_1}}{\frac{\partial v^L}{\partial \delta_2}} = \frac{(1-p^L)u'(w+\alpha_1)}{p^L\,u'(w-d+\alpha_2)} > \frac{(1-p^H)u'(w+\alpha_1)}{p^H\,u'(w-d+\alpha_2)} = \frac{\frac{\partial v^H}{\partial \delta_1}}{\frac{\partial v^H}{\partial \delta_2}} \equiv mrs^H$$

i.e., the low risk indifference curve is more steeply sloped at α than the high risk indifference curve. Hence there is a contract β such that $v(p^L, \beta) > v(p^L, \alpha)$ and $v(p^H, \alpha) > v(p^H, \beta)$. It follows that there exists a contract β such that $\beta \in \Delta^+(p^L)$. Since β is selected so that $v(p^L, \beta) > v(p^H, \alpha)$, all low risk individuals switch to contract β. This makes α lose because $\alpha \in \Delta^+(p^L)$ This is shown in the following figure.

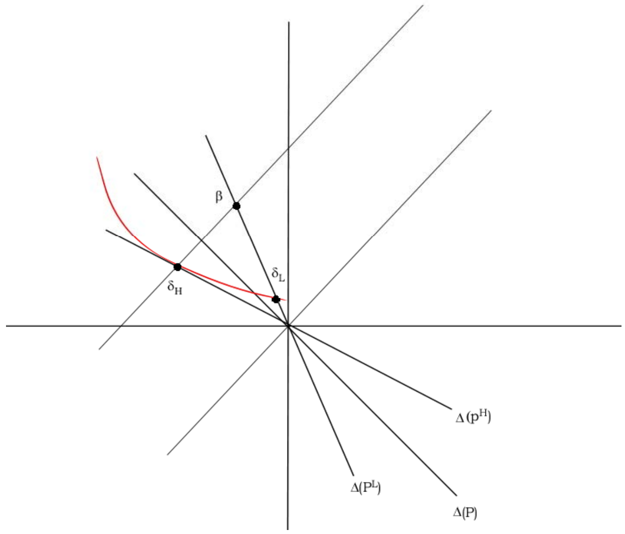


Separating Equilibria

Each contract in the equilibrium set must earn zero profits. The low risk individuals must purchase a contract δ such that $\pi(p^L, \delta) = 0$ and the high risk a contract δ such that $\pi(p^H, \delta) = 0$. Recall that

$$\frac{1-p^L}{p^L} > \frac{1-p^H}{p^H}$$

Next consider the possibility of a separating equilibrium.

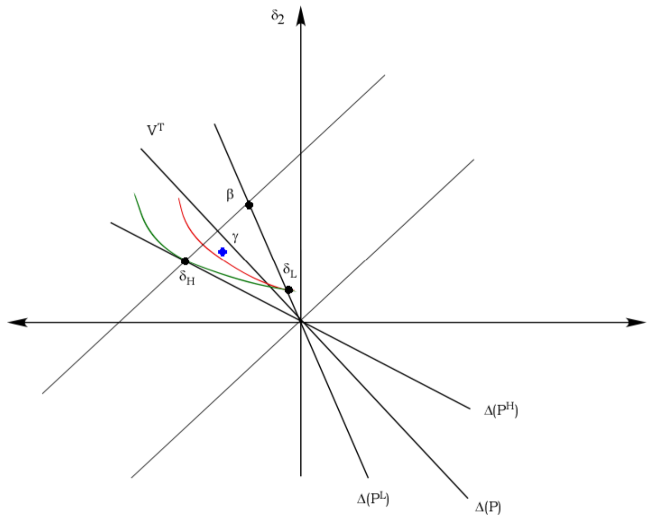


Any contract on $\Delta\left(p^{H}\right)$ northwest of δ^{L} would yield $v\left(p^{H}, \delta\right) > v\left(p^{H}, \delta^{H}\right)$ and so cannot be used to separate low from high risk. Since δ^{L} is the contract on $\Delta\left(p^{L}\right)$ most preferred by the low-risk individuals, the pair $\left(\delta^{L}, \delta^{H}\right)$ is a separating equilibrium if one exists.

A pooling contract γ can be used, in some cases to break the separating equilibrium if the following conditions are met:

1. $\pi\left(\overline{p}, \gamma\right) > 0$
2. $v\left(p^{H}, \gamma\right) > v\left(p^{H}, \delta^{H}\right)$
3. $v\left(p^{L}, \gamma\right) > v\left(p^{L}, \delta^{L}\right)$

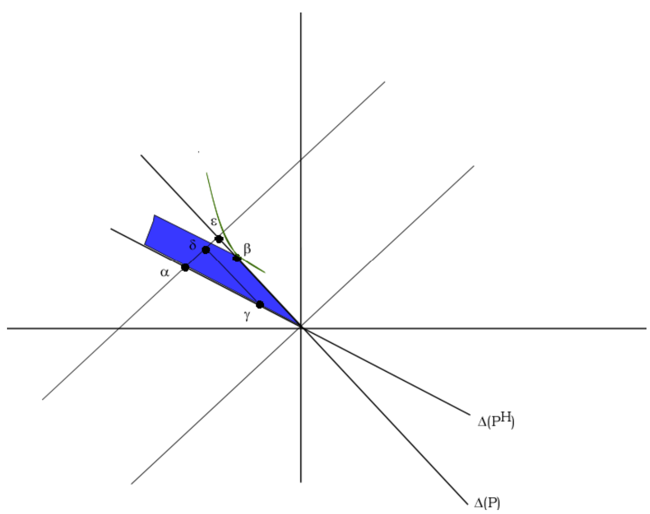
These conditions are shown in the next figure.



Therefore, the pooling contract γ prevents (δ^L, δ^H) from being a separating equilibrium.

Nash Equilibria

There are some problems with the Nash equilibrium concept here. In particular, there must be some implicit or explicit cooperation between insurance firms to enforce the one contract restriction imposed on consumers. If that restriction is lifted then a different kind of equilibrium contract pair appears as Nash equilibrium. The new Nash equilibrium always exists. Also note that a pooling contract exists in this equilibrium pair. The following figure depicts the equilibrium.



Let $\delta = \beta + \gamma$

$$\frac{\frac{\partial v^L}{\partial \delta_1}}{\frac{\partial v^L}{\partial \delta_2}} = \frac{(1 - p^L)}{p^L} > \frac{(1 - \bar{p})}{\bar{p}}$$

at the contract ε. Hence, given a pooling contract, the low risk group will not select full insurance.

An equilibrium pair (β, γ) exists if there is no quantity setting and firms do not share information. The high risk individuals will purchase $\delta = \beta + \gamma$ while the low risk buy □, i.e. complete and incomplete respectively. Note that the same procedure as above cannot be used to break any potential pair as a separating equilibrium.