Stock Options and Capital Structure

by

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0. Introduction

The empirical evidence on sources of corporate financing strongly suggests that firms prefer internally generated funds to debt and debt to equity in financing their investment activities (see for example table 14-3 in Brealey and Myers (1991)). Additional evidence of this preference ordering over financing methods can be found in Agrawal and Mandelker (1987), Hittle, Haddad et al. (1992), Jensen, Solberg et al. (1992), and Tamule, Bubnys et al. (1993). What is the economic rationale for this preference ordering? Explanations in the existing literature rest upon informational asymmetries between corporate managers and investors and the agency costs these asymmetries generate. For example, (Myers and Majluf 1984) argue that if investors are less well-informed than the corporate manager about the value of the firm's assets, then new equity may be under-priced by the market. If firms are required to finance projects by issuing equity, under-pricing may be so severe that new investors capture more than the net present value of the project, resulting in a net loss to existing shareholders. In this case, the project will be rejected even if its net present value is positive. This under-investment problem can be avoided if the firm can finance the new project using a security that is not so severely undervalued by the market. For example, because internally generated funds or safe debt generate no agency cost, these methods of financing involve no under-valuation, and therefore will be preferred to equity by firms in this situation. Myers refers to this as a *pecking order* theory of financing. One problem with this theory is that it does not explain the apparent preference of corporate managers for internally generated over *all* external sources of funds, including safe debt..

An even more fundamental problem with pecking order theories based on asymmetric information and agency costs is that the conclusions of these theories are not robust. Thus, the conclusions of these theories are sometimes at odds with the consistent empirical evidence supporting a pecking order. Despite the simplicity and appeal of Myers and Majluf's arguments, further theoretical analyses by Brennan and Kraus (1987), Noe (1988), and (Constantinides and Grundy 1989) have cast doubt on the Myers and Majluf explanation of the pecking order.[1] These papers conclude that if the set of financing choices available to the firm is expanded, then when faced with the situation considered by Myers and Majluf, firms do not necessarily prefer issuing straight debt over equity. Moreover, these papers show that with the richer set of financing options, firms can resolve the under-investment problem through costless signaling.

In this paper we provide an alternative explanation for the prevalence of a corporate pecking order over financing methods. More importantly, we put the *pecking order theory* of capital structure on a different, and we believe, stronger theoretical foundation. Myers and Majluf assume that the financing decision is made to maximize current shareholder value. Given less well-informed outside investors, the consequent agency costs, and a manager acting in the interests of current shareholders, the pecking order theory follows naturally and explains how the agency cost can be minimized. There are, however, two sources of difficulty in the Myers and Majluf approach. The first has been demonstrated by Brennan and Kraus, Constantinides and Grundy, and Noe: if the set of financing options is expanded, then in some cases the manager can reveal information to the outsiders costlessly, thus avoiding any agency cost. The second difficulty is more fundamental: in Myers and Majluf, as well as in much of the existing literature, the objective function, used by the manager in making decisions on behalf of the corporation, is *assumed* rather than derived.[2] In this paper, we endogenously derive

[1]Harris and Raviv (1991) also provide a discussion of this literature.

[2] There is a very small literature related to the corporate objective function. John and John (1993) take a given capital structure and ask what the optimal executive compensation package is. Brander and Poitevin

the manager's objective function and show how it depends on the form of the manager's compensation contract. In order to give a concrete example of this connection, we derive the manager's objective function for the case in which the manager's compensation includes stock options. This case is important because nearly 90% of Fortune 500 companies offer potentially lucrative incentives to their executives including stock options.[3] We then analyze the implications of the stock-option-based objective function for optimal capital structure decisions by the manager. In particular, we show that if the manager's compensation includes stock options with a positive exercise price, then the manager's objective function is such that the manager prefers internally generated funds to debt, and debt to equity in financing the firm's investments. Thus, by directly addressing the issue of executive compensation and managerial objectives, we provide a compensation-based explanation for the pecking order theory of capital structure.

The analysis here is a partial equilibrium analysis in two important respects. First, we only consider the agent's problem. In an expanded treatment of the *principal-agent* problem one would have to consider, from the perspective of the principal, the rationale for including stock options in the executive compensation package. Why is this form of compensation so pervasive? In MacMinn (1995) this question is addressed. MacMinn (1995) considers a firm which operates simultaneously in a competitive financial market but an *imperfectly competitive* product market and demonstrates that, from the perspective of the board of directors (who collectively act as the principal and represent the interests of shareholders), it is optimal to select a managerial compensation package that includes stock options *because such a compensation package strategically*

(1992) consider the manager's objective function given a compensation scheme that includes a bonus. There is also a much older thread to this literature in Ekern and Wilson (1974 and Radner (1974) .

[3]See Joann S. Lubin, "The American Advantage," *Wall Street Journal*, Wednesday, April 17, 1991, R4.

commits the manager to a stronger course of action against the firm's competitors in the product market. Alternatively, if the product market in which the firm operates is competitive, then the inclusion of stock options is no longer optimal. Thus, the presence of imperfectly competitive product markets provides the rationale for the pervasive use of stock options in executive compensation.

There is another respect in which the analysis here is a partial equilibrium analysis. In valuing the firm's capital structure, the manager uses a vector of state-contingent claims prices. Thus, the manager values the firms capital structure *as if* markets were complete. One important feature of our analysis is that our conclusions regarding stock options and capital structure do not depend on the specific vector of state-contingent claims prices used by the manager. In particular, no matter what vector of state-contingent claims prices is used by the manager in valuing the firm's capital structure, the conclusion is the same: the manger prefers internally generated funds to debt, and debt to equity. Thus, our conclusions are robust with respect to market incompleteness.

The paper is organized as follows: In section one, the financial market model is constructed and used to value bond and stock contracts. In section two, the stock option contract is introduced and used to derive the manager's objective function. There we show that the manager makes decisions on corporate account to maximize the value of the stock option package. In section three, we allow the manager to make financing decisions and show that the manager prefers internal to external financing, all debt to all equity, and all debt to any combination of debt and equity. In section four, we provide some concluding remarks and conjectures.

1. Financial Markets

Consider a competitive financial market operating between dates *now* and *then*. Consumers make portfolio decisions *now* that determine saving, or equivalently, consumption *now* and *then*. Corporations make investment and financing decisions *now* that determine the return on financial instruments *then*. Given risky investments, this analysis is a generalization of the classic Fisher model.

1.1. Notation and Assumptions

t = 0, 1; t indexes dates *now* and *then*, respectively

(Ω, F, λ) = state space

P = manager's probability measure over states

$u(\cdot, \cdot)$ = manager's utility function over $R \times R$

m_0 = income *now*

m_1 = income *then*

c_0 = consumption *now*

$c_1(\omega)$ = consumption *then* in state $\omega \in \Omega$

I_0 = investment *now*

d_0 = dividend payment *now*

π_0 = firm's earnings *now*

$\Pi_1(\omega)$ = firm's earnings *then* given state ω, and investment I_0

N = the number of common stock before any new issue *now*

m = the number of new shares of common stock issued *now*

b = the promised payment to bondholders *then*

[A-1] The manager's utility function is concave, increasing, and everywhere differentiable.

[A-2] The probability measure P is absolutely continuous with respect to λ, i.e., $P \ll \lambda$.

[A-3] The corporate payoff function is non-negative, i.e., $\Pi_1(\omega) \geq 0$ for all $\omega \in \Omega$.

1.2 Value

Let $L_\infty(\lambda)$ denote the set of all λ-essentially bounded, real-valued, F-measurable functions defined on Ω, and let $\upsilon(\cdot) \in L_\infty(\lambda)$.

[A-4] (a) For any $E \in$ F, an asset with a payoff of one dollar for any $\omega \in E$ and zero otherwise, has a market value

$$\int_E \upsilon(\omega)\, d\lambda(\omega) \geq 0.$$

(b) $\int_\Omega \upsilon(\omega)\, d\lambda(\omega) = \frac{1}{1 + r}$, where r is the rate of return on a safe asset.

Define a zero coupon bond contract as a promise to pay one dollar *then*. In the absence of any stock options, a common stock is a promise to pay d_0/N dollars *now* to holders of record and $\max\{0, \Pi - b\}/(N + m)$ dollars *then*.[4] Suppose an investor holds x^o bonds and y^o stocks *now*. Suppose the investor has an income pair (m_0, m_1), and that the corporate bond issue is safe. In its classic form, the Fisher model specifies the investor's problem as a constrained maximization problem in which the investor selects

[4]Since the firm selects a dividend *now* as well as raising funds to cover its investment of I_0 dollars, the sequence of events must be specified. Here we suppose that the firm makes its dividend payment to the N holders of record and then issues new securities. All stock prices calculated here are therefore ex-dividend prices.

consumption *now* and *then* to maximize expected utility subject to a budget constraint. By [A-4], the investor's problem is

$$\text{maximize} \int_\Omega u(c_0, c_1(\omega))\, dP(\omega)$$

$$\text{subject to } c_0 + \int_\Omega c_1(\omega)\, \upsilon(\omega)\, d\lambda(\omega) = m_0 + p_b\left(x^o - x\right) + p_s\left(y^o - y\right) + y^o \frac{d_0}{N}$$

$$+ \int_\Omega m_1\, \upsilon(\omega)\, d\lambda(\omega) + x \int_\Omega \upsilon(\omega)\, d\lambda(\omega) + y \int_\Omega \frac{\Pi_1(\omega) - b}{N + m}\, \upsilon(\omega)\, d\lambda(\omega)$$

The budget constraint sets the risk adjusted present value of consumption equal to the risk adjusted present value of income. If the investor liquidates the bond position then x equals zero and $p_b\, x^o$ dollars are raised *now*. Similarly, if the investor holds on to all the bonds then no dollars are raised *now* from bond sales. The first order conditions, or equivalently, the no arbitrage conditions, yield the bond and stock prices *now* as

$$p_b = \int_\Omega \upsilon(\omega)\, d\lambda(\omega) = \frac{1}{1 + r}$$

and

$$p_s = \int_\Omega \frac{\Pi_1(\omega) - b}{N + m}\, \upsilon(\omega)\, d\lambda(\omega)$$

Note that the value of bond issue is D(b) = b/(1 + r) and the value of the stock issue is

$$S(b, m) \equiv p_s\left(N + m\right) = \int_\Omega \left(\Pi_1(\omega) - b\right)\upsilon(\omega)\, d\lambda(\omega) \qquad (1)$$

Alternatively, for any $E \in \mathcal{F}$ let,

$$\mu(E) = \int_E (1 + r)\, \upsilon(\omega)\, d\lambda(\omega) \tag{2}$$

Given [A-4], μ is a probability measure and for $\Pi(\cdot) - b \in L_1(\lambda)$

$$\int_\Omega \left(\Pi_1(\omega) - b\right) d\mu(\omega) = \int_\Omega \left(\Pi_1(\omega) - b\right) (1 + r)\, \upsilon(\omega)\, d\lambda(\omega)$$

Thus, the stock market value *now* can be equivalently expressed as

$$S = \frac{1}{1 + r} \int_\Omega \left(\Pi_1(\omega) - b\right) d\mu(\omega) \tag{3}$$

2. Stock Options and the Manager's Objective Function

In the economics literature, agents are assumed to pursue their self interests subject to constraints. Given uncertainty, this behavior is often specified as a constrained expected utility maximization problem. The theory of the firm is an exception. The fictitious agent known as the firm is assumed to maximize profit. Under uncertainty, the theory of the firm has been specified in an expected utility framework (Baron 1970; Sandmo 1971; Leland 1972; Stiglitz and Greenwald 1990) but the fictitious agent is assumed to maximize the expected utility of profit. These theories of the firm have not been derived from the more primitive notion of the pursuit of self interest.

Here the manager is given the role of an investor as well as that of a manager and assumed to make portfolio decisions on personal account and financing decisions on corporate account in the pursuit of self interest. The analysis here shows that the objective function of the firm may be derived rather than assumed. The corporate objective function, or equivalently, the objective function that the manager uses to make decisions on behalf of the corporation, flows from a Fisher Separation result. As with all Fisher Separation results, the decisions made on corporate account are independent of the manager's probability and risk aversion measures; unlike other Fisher Separation results, the analysis here shows that the manager does not make decisions that maximize the risk adjusted present value of the shareholders' stake in the corporation. Equivalently, this analysis shows that the manager does not make decisions to maximize the current shareholder value.

Since stock options are an important component of management compensation schemes, we assume that the manager is paid a safe salary *now* and *then* and receives a stock option grant *now*.[5] Stock options are simply call options or warrants that give their owners the option to purchase a certain number of stocks *then* at an exercise price that is specified *now*. That exercise price is typically the common stock price on the issue date. An option is "in the money" *then* if the exercise price is less than the stock price; otherwise it is "out of the money." The exercise event defined in the subsequent analysis is equivalent to the *in the money* event.

2.1. Additional Notation

e = the exercise price *then* on each stock option

n = the number of stock options issued to the manager *now*

p_w = the stock option or warrant price *now*

$W \equiv p_w n$, the value of the warrant package *now*

[5]It is possible to allow some trading in options *now* without affecting the results that follow as long as the manager must maintain some positive position in the stock options. There is, however, a period of several years before the options in a particular grant can be exercised.

2.2. Stock Options and Financial Market Values

The manager's compensation package includes a safe salary *now* and *then*, (m_0, m_1), and n stock options with an exercise price of e dollars per option. The payoff on each option is given by

$$\max\left\{0, \frac{\Pi_1(\omega) + e\,n - b}{N + m + n} - e\right\} \tag{4}$$

Let $E(b, e, m) = \{\omega \in \Omega \mid \Pi_1(\omega) \geq b + e\,(N + m)\}$ be the exercise event, or equivalently, the event that the stock option is in the money. The price of the stock option, equivalently, warrant, is

$$p_W(b, e, m) = \int_{\Omega} \max\left\{0, \frac{\Pi_1(\omega) + e\,n - b}{N + m + n} - e\right\} \upsilon(\omega)\, d\lambda(\omega)$$

$$= \int_{E(b,\, e,\, m)} \left(\frac{\Pi_1(\omega) + e\,n - b}{N + m + n} - e\right) \upsilon(\omega)\, d\lambda(\omega) \tag{5}$$

Letting $\xi \equiv n/(N + m + n)$ denote the proportion of the firm acquired by the manager upon exercising the options, it follows that the option or warrant value is

$$W(b, e, m) = \int_{E(b,\, e,\, m)} \left(\xi\left(\Pi_1(\omega) + e\,n - b\right) - e\,n\right) \upsilon(\omega)\, d\lambda(\omega)$$

$$= \int_{E(b,\, e,\, m)} \left(\xi\left(\Pi_1(\omega) - b\right) - (1 - \xi)\, E\right) \upsilon(\omega)\, d\lambda(\omega) \tag{6}$$

where $E \equiv e\,n$.

Next, let

$$B(b) = \{\omega \in \Omega \mid \Pi_1(\omega) < b\} \tag{7}$$

denote the bankruptcy event and

$$N(b, e, m) = \{\omega \in \Omega \mid b \le \Pi_1(\omega) \le b + e\,(N + m)\} \tag{8}$$

denote the no bankruptcy and no exercise event. The aggregate debt value is

$$D(b) = \int_{B(b)} \Pi_1(\omega)\, \upsilon(\omega)\, d\lambda(\omega) + \int_{\Omega \setminus B(b)} b\, \upsilon(\omega)\, d\lambda(\omega) \tag{9}$$

and the common stock price is

$$p_s(b, e, m) = \int_{N(b,\, e,\, m)} \frac{\Pi_1(\omega) - b}{N + m}\, \upsilon(\omega)\, d\lambda(\omega) + \int_{E(b,\, e,\, m)} \frac{\Pi_1(\omega) + e\,n - b}{N + m + n}\, \upsilon(\omega)\, d\lambda(\omega) \tag{10}$$

Similarly, the stock value is the stock price times the number of shares available *now*, i.e., $p_s\,(N + m)$ and

$$S(b, e, m) \equiv p_s(b, e, m) \left(N + m\right)$$

$$= \int_{N(b,\, e,\, m)} \left(\Pi_1(\omega) - b\right) \upsilon(\omega)\, d\lambda(\omega) \tag{11}$$

$$+ \int_{E(b,\, e,\, m)} \left(1 - \xi\right) \left(\Pi_1(\omega) + e\,n - b\right) \upsilon(\omega)\, d\lambda(\omega)$$

Observe that the corporate value V that is defined as the sum of the debt, equity, and warrant values, i.e., $V \equiv D + S + W$, is

$$V = D(b) + S(b, e, m) + W(b, e, m) = \int_{\Omega} \Pi_1(\omega)\, \upsilon(\omega)\, d\lambda(\omega) \tag{12}$$

This is a generalized version of the 1958 Modigliani-Miller Theorem (Modigliani and Miller 1958). Like the Modigliani-Miller theorem this result shows that corporate value is independent of the structure of the financing decisions, i.e., the investment can be financed with any combination of debt or equity without affecting corporate value. It also shows that corporate value is independent of the provisions of the stock option package, in particular, the exercise price.

2.3. The Manager's Objective Function

The manager makes decisions on personal and corporate account to maximize her own self interest. Suppose the manager's feasible set of consumption possibilities *then* is $\Lambda = \{c_1(\cdot) \mid c_1(\cdot) \text{ is } \mathcal{F}\text{-measurable and } 0 \le c_1(\omega) \le \zeta_1(\omega) \text{ a.e. } [\lambda]\}$, where $\zeta_1(\omega) \in L_1(\lambda)$. The problem on personal account is the selection of $(c_0, c_1(\cdot)) \in R_+ \times \Lambda$. The problem on corporate account is the selection of (b, m) such that $\pi_0 + D(b) + p_s(b, e, m)\, m = I_0 + d_0$.

D(b) represents the dollar amount raised with a new bond issue while p_s m represents the amount raised with a new stock issue. The manager's problem may be expressed as

$$\text{maximize} \int_\Omega u(c_0, c_1(\omega))\, dP(\omega)$$

$$\text{subject to } c_0 + \int_\Omega c_1(\omega)\, \upsilon(\omega)\, d\lambda(\omega) = m_0 + W(b, e, m) + \int_\Omega m_1\, \upsilon(\omega)\, d\lambda(\omega) \quad (13)$$

$$\text{and } \pi_0 + D(b) + p_s(b, e, m)\, m = I_0 + d_0$$

The equations in (13) represent the budget and the financing constraints. It is easy to see that in choosing a pair (b, m) for the firm, the manager will seek to maximize the value of the stock option package W. Hence, the manager will make decisions on corporate account to

$$\text{maximize } W(b, e, m)$$

$$\text{subject to } \pi_0 + D(b) + p_s(b, e, m)\, m = I_0 + d_0 \quad (14)$$

From (6), (9), and (10) it follows that this is a Fisher Separation result. The objective function in (14) does not, however, generate a unanimity result and it is not consistent with the standard assumption in the finance literature that managers maximize current shareholder value. The objective function here does provide some insight into how and why financing decisions may be relevant.

3. Financing Decisions

Given the objective function in (14), the corporate manager may choose to finance the investment with retained earnings, debt, equity, or some combination of the three. We consider the retained earnings versus debt, the debt versus equity, and then generalize to allow for any combination.

Let $\mu(\cdot)$ be the market pure probability measure induced by the value operator $\upsilon(\cdot)$ and let $F(\cdot)$ be a risk adjusted probability distribution induced by $\Pi_1(\cdot)$. Then

$$F(\pi_1) = \mu\{\omega \mid \Pi_1(\omega) \leq \pi_1\}$$

Note that F is not the manager's probability distribution.[6] Rather, F is a risk adjusted probability distribution that embeds the probability and risk aversion measures of all agents through the value operator $\upsilon(\cdot)$.

[A-5] $F(\cdot)$ has a continuous density $f(\cdot)$: $[0, \infty) \rightarrow R_+$ such that $\lim_{\pi \rightarrow \infty} \pi_1 f(\pi_1) = 0$.

Given [A-5], the value of the manager's stock option package that is specified in (6) may be equivalently expressed as

$$W(b, e, m) = \frac{1}{1 + r} \int_{b+e(N+m)}^{\infty} \left(\frac{n}{N + m + n} \left(\pi_1 + en - b\right) - en \right) f(\pi_1)\, d\pi_1 \qquad (16)$$

3.1 Retained Earnings versus Debt

Suppose the manager finances the investment with retained earnings or debt. The financing constraint is $D(b) = I_0 - (\pi_0 - d_0)$.. Since the debt value is an increasing function of the promised payment b, it follows that inceasing the use of retained earnings decreases the necessary promised payment. Consider the following two cases:

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[6]The manager's distribution function would be a function $G(\pi_1) = P\{\omega \mid \Pi_1(\omega) \leq \pi_1\}$.

Case I: Let the dividend d_0 be zero. Equivalently, let the retained earnings be π_0. If $\pi_0 \leq I_0$ then the payment on a bond issue is determined by the condition $D(b_1) = I_0 - \pi_0$. In this case the stock option payoff is $\max\{0, \iota\ (\Pi_1 - b_1) - (1 - \iota)\ E\}$, where $\iota \equiv n/(N + n)$.

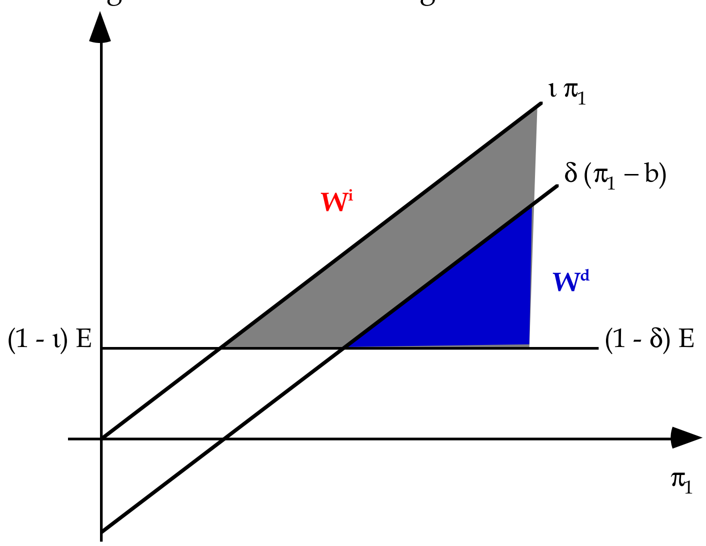
Case II: Let the dividend be equal to the earnings *now*, i.e., $d_0 = \pi_0$. Suppose the firm uses a debt issue so that $D(b_2) = I_0$. In this case the stock option payoff is $\max\{0, \delta\ (\Pi_1 - b_2) - (1 - \delta)\ E\}$, where $\delta = n/(N + n)$.

Note that $\iota \equiv \delta$. Since D(b) is increasing in b, a comparison of the two cases makes it clear that the stock option value given internal finance is greater than that given a debt issue, even if the debt issue is safe! This result is stated in the following proposition and demonstrated in figure one for the case in which $\pi_0 = I_0$.

Proposition 1. Given the stock option compensation scheme, the manager prefers retained earnings, or equivalently, internal equity, to debt in financing an investment.

This result is different than that of Myers and Majluf. The Myers and Majluf analysis shows that retained earnings and safe debt are equivalent in a pecking order. Since Myers and Majluf motivated the pecking order with the asymmetric information assumption, the equivalence of these two forms of financing follows easily because neither form of financing is sensitive to the difference between insider and outsider information; hence, neither form of financing generates an agency cost that determines a position in the pecking order.

Figure 1: Retained Earnings versus Bonds



The result in proposition one is not motivated by an asymmetric information assumption. Rather, it depends on the assumption that the manager pursues her self interest. By using retained earnings to finance the investment rather than debt, the manager generates greater earnings *then* and so a greater stock price *then* when the options can be exercised. It should also be noted that if the exercise price is set equal to zero then the warrants become stock but the proposition still holds.

3.2 Debt versus Equity

Next, suppose that the investment I_0 exceeds the firm's ability to generate funds internally; let $I_0 - \pi_0 > 0$ and consider the case of debt versus equity financing. By hypothesis in this section, the manager selects either debt or equity to finance the $I_0 - \pi_0$ dollars. If the manager maximized current shareholder value subject to the financing constraint then the manager would be indifferent to the choice of debt versus equity. The manager who maximizes the value of a stock option package subject to the financing constraint, however, is not indifferent, because the warrant values are, in part, determined by the capital structure of the firm.

First, consider the case of debt finance. If the I_0 - π_0 dollars is obtained with a debt issue then the financing constraint is

$$D(b) = I_0 - \pi_0 \tag{17}$$

where

$$D(b) = \frac{1}{1 + r} \int_0^b \pi_1 \ f(\pi_1) \ d\pi_1 + \frac{1}{1 + r} \int_b^\infty b \ f(\pi_1) \ d\pi_1 \tag{18}$$

Letting $\delta = n/(N + n)$, the value of the manager's stock option package with all debt financing is given by[7]

$$W(b, e, 0) = \frac{1}{1 + r} \int_{b+e\,N}^\infty \left(\delta \left(\pi_1 - b\right) - \left(1 - \delta\right) e\,n\right) f(\pi_1) \ d\pi_1 \tag{19}$$

Next, suppose the required investment amount of I_0 - π_0 dollars is obtained with an equity issue. In this case, the financing constraint is

$$p_s(0, e, m) \ m = I_0 - \pi_0 \tag{20}$$

where

[7]In the subsequent analysis, we will use the terms all debt and all equity to refer to the external financing, i.e., all debt means that all the external funds required are raised with a debt issue.

$$p_s(0, e, m)\, m = \frac{1}{1+r}\int_0^{e(N+m)} \frac{m}{N+m}\, \pi_1\, f(\pi_1)\, d\pi_1 + \frac{1}{1+r}\int_{e(N+m)}^{\infty} \frac{m}{N+m+n}\left(\pi_1 + e\,n\right) f(\pi_1)\, d\pi_1 \tag{21}$$

It follows from the financing constraint (20) that the number of new shares that must be issued to raise $I_0 - \pi_0$ dollars, i.e., m, is an implicit function of the exercise price e. Letting $\xi(e) = n/(N + m(e) + n)$, the value of the manager's stock option package with all equity financing is given by

$$W(0, e, m(e)) = \frac{1}{1+r}\int_{e(N+m(e))}^{\infty} \left(\xi(e)\, \pi_1 - \left(1 - \xi(e)\right) e\,n\right) f(\pi_1)\, d\pi_1 \tag{22}$$

The following figures provide a comparison of the warrant payoffs in the debt and equity finance cases. The warrant payoff given a debt issue is $\max\{0, \delta\,(\pi_1 - b) - (1 - \delta)\, e\, n\}$ while the warrant payoff given an equity issue is $\max\{0, \xi(e)\, \pi_1 - (1 - \xi(e))\, e\, n\}$. Note that $\delta > \xi(e)$ and that this shows that a debt issue causes less dilution in the warrant payoff. Given the separation result, the larger proportional stake in the corporate payoff given a debt issue suggests the basis for a preference of debt over equity. If the *in the money* event, or equivalently, the exercise event is also larger for the debt issue then the debt issue clearly dominates the equity issue.

Figure 2: *Warrant Values - Case I*

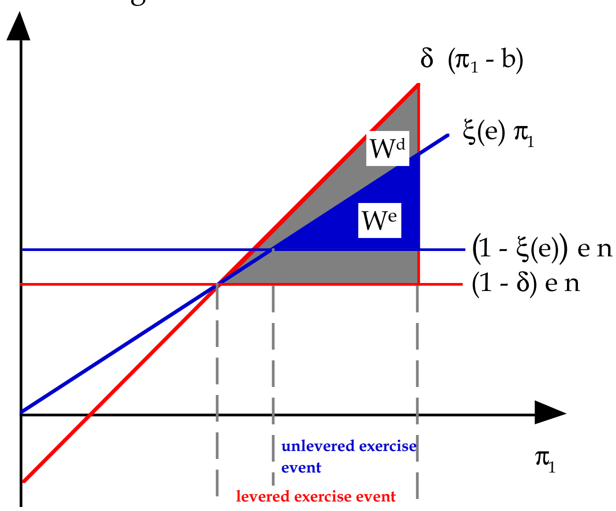


Figure two depicts the case in which debt is clearly preferred by the manager. The values of the shaded areas labeled W^e and W^d represent the warrant values in the equity and debt issue cases, respectively. The lower bound π_1^e of the unlevered exercise event is implicitly defined by the condition $\xi(e)\ \pi_1^e = (1 - \xi(e))\ e\ n$, while the lower bound π_1^d of the levered exercise event is implicitly defined by the condition $\delta\ (\pi_1^d - b) = (1 - \delta)\ e\ n$. In the case depicted in figure two, $\pi_1^e > \pi_1^d$, or

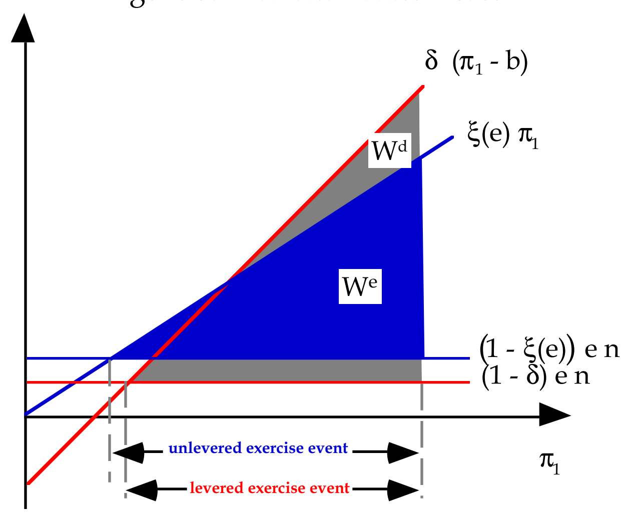
$$\frac{1 - \xi(e)}{\xi(e)}\ e\ n > \frac{(1 - \delta)\ e\ n + \delta\ b}{\delta} \tag{23}$$

or equivalently, $e\ (N + m) \geq b + e\ N$. It follows that $\max\{0, \delta\ \pi_1 - (1 - \delta)\ e\ n - \delta\ b\} \geq \max\{0, \xi(e)\ \pi_1 - (1 - \xi(e))\ e\ n\}$, for all π_1 and strictly greater for some π_1. If the probability that the option is in the money is positive, i.e., $1 - F(b + e\ N) > 0$, then

$$W(b, e, 0) > W(0, e, m(e)) \tag{24}$$

The inequality in (23) holds if and only if e m(e) > b. For the other cases in which b > e m(e), it is not clear whether or not (24) holds. The case b > e m(e) is depicted in figure three.

Figure 3: *Warrant Values - Case II*



The following proposition shows that (24) does hold generally. We suppose that the payoff is positive with probability one and that the exercise event is risky. If b > e m(e) and F(e m(e)) > 0 for positive exercise prices then the probability of exercise is larger for the equity issue than it is for the debt issue since 1 - F(b) < 1 - F(e m(e)). Hence, both exercise events are risky.

Proposition 2. Let F(0) = 0, 0 < F(b + e N) < 1 and 0 < F(e (N + m)) < 1 for e > 0. Then the manager prefers a debt issue to an equity issue given any positive exercise price.

Proof. It suffices to show that (24) holds whenever b > e m(e). Consider the following argument. Let b be the payment such that

$$D(b) = \frac{1}{1 + r}\left\{\int_{-\infty}^{b} \pi_1\ f(\pi_1)\ d\pi_1 + \int_{b}^{\infty} b\ f(\pi_1)\ d\pi_1\right\} = I_0 - \pi_0$$

Now, suppose that e = 0. Then

$$W(b,0,0) - W(0, m(0), 0) = \frac{1}{1 + r}\left\{\int_{-\infty}^{+\infty} \left(\max\left\{0, \delta(\pi_1 - b)\right\} - \xi(0)\ \pi_1\right) f(\pi_1)\ d\pi_1\right\}$$

$$= \frac{1}{1 + r}\left\{\int_{-\infty}^{+\infty} \left(\delta\left(\pi_1 - \min\left\{\pi_1, b\right\}\right) - \xi(0)\ \pi_1\right) f(\pi_1)\ d\pi_1\right\}$$

$$= \delta\left(V - (I_0 - \pi_0)\right) - \xi(0)\ V$$

Now, $\delta\left(V - (I_0 - \pi_0)\right) = \xi(0)\ V$ if and only if

$$\frac{\delta - \xi(0)}{\delta} = \frac{I_0 - \pi_0}{V} \tag{25}$$

If the exercise price is zero then m(0) is determined by $p_s(0, 0, m(0))\ m = I_0 - \pi_0$. Note that e = 0 yields the exercise event $E(0, 0, m(0)) = \Omega$ and the no bankruptcy and no exercise event $N(0, 0, m(0)) = \phi$, i.e., the empty set. Hence, by (9), $p(0, 0, m(0))\ m = V/(N + n + m)$. It follows that

$$\frac{I_0 - \pi_0}{V} = \frac{m(0)}{N + n + m(0)}$$

and so (25) becomes

$$\frac{\delta - \xi(0)}{\delta} = \frac{m(0)}{N + n + m(0)}$$

But now

$$\frac{\delta - \xi(0)}{\delta} = \frac{\frac{n}{N+n} - \frac{n}{N + n + m(0)}}{\frac{n}{N+n}} = 1 - \frac{N+n}{N + n + m(0)} = \frac{m(0)}{N + n + m(0)}$$

Therefore (25) holds and W(b, 0, 0) - W(0, 0, m(0)) = δ (V - I) - ξ(0) V = 0.

Next, we want to consider

$$\frac{\partial}{\partial e}\Big(W(b, e, 0) - W(0, e, m(e))\Big)$$

for the case b > e m(e). If b > e m(e) then

$$\frac{\partial}{\partial e}\Big(W(b, e, 0) - W(0, e, m(e))\Big) > \frac{1}{1+r}\int_{e\left(N+m(e)\right)}^{b+eN} \frac{N}{N+n}\, n\, f(\pi_1)\, d\pi_1 \geq 0^{8} \qquad (26)$$

Thus W(b, 0, 0) - W(0, 0, m(0)) = 0 and as e increases so does W(b, e, 0) - W(0, e, m(e)), as long as b > e m(e). Since we also know that W(b, e, 0) - W(0, e, m(e)) > 0 for b ≤ e m(e) and e > 0, the result holds for all positive exercise prices. Q.E.D.

The classic preference result in the literature is an indifference to the form of financing. The method of proof employed here captures that result by showing that if

[8] These inequalities are established in the appendix.

the exercise price on the stock options is zero then the value of the warrant package is the same for debt and equity finance. The proposition, however, also demonstrates a managerial preference for debt over equity if the stock option package is risky, i.e., has a probability of exercise less than one. The proposition goes beyond the results available in the option pricing literature because the stock options, or equivalently, warrants considered here are part of the firm's capital structure.[9]

3.3 The Optimal Mix of Debt and Equity

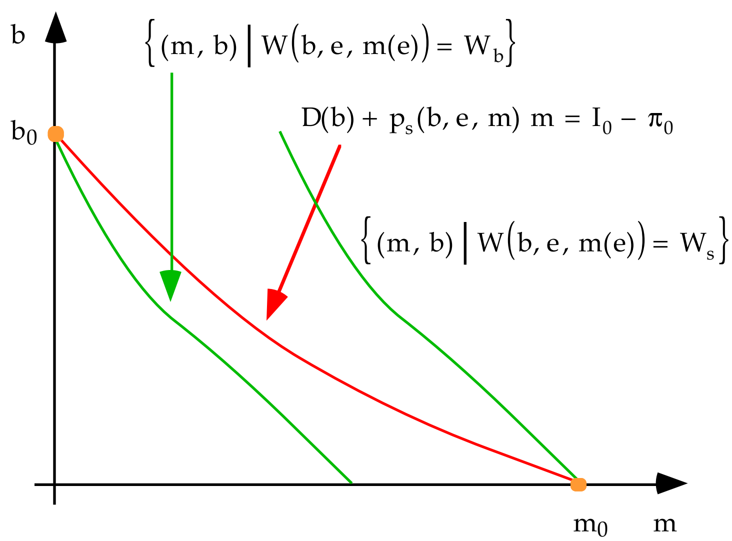
If the manager is restricted to either a debt or equity choice then the analysis here shows that the manager has the incentive to select a debt issue. Such a choice yields a larger stock option value now. The constrained maximization problem in (14) is, however, more general. The manager may select a combination of the two in financing the firm's investment. The purpose of this section is to generalize the previous section's choice analysis by allowing the manager to select any combination of debt and equity that satisfies the financing constraint.

The constrained maximization problem in (14) is a similar to many choice problems in economics. There are two difference here. First, the objective function W is endogenous, i.e., due to a Fisher Separation result. Second, smaller values of b and m are preferred and so the iso-value contours of the objective function closer to the origin represent larger warrant values. The level sets $\left\{(m,b)\,\middle|\,W\big(b,e,m(e)\big)=\overline{W}\right\}$ represent iso-warrant values. Let $W_s \equiv W(0, e, m_0(e))$ and $W_b \equiv W(b_0, e, 0)$, where b_0 and $m_0(e)$ finance

[9]The result in the option pricing literature is that an increase in risk yields an increase in call option value Merton, R. C. (1973). "Theory of Rational Option Pricing." Bell Journal of Economics and Management Science **4**(1): 141-83.. Although one might loosely argue that levering the firm increases risk, a straight forward application of Merton's theorem eight to the analysis here is inappropriate. Levering the firm can be shown to cause an increase in risk but that increase is in a value preserving sense rather than the mean preserving sense that Merton required. In addition, the analysis here is for warrants rather than the call options that Merton considered. A call option is not issued by the firm whereas the warrant is.

the investment in the debt and equity case, respectively. By proposition two, $W_b > W_s$. The following figure depicts the iso-warrant value curves for W_b and W_s. If the iso-warrant lines are steeper everywhere than the finance constraint then any movement from the all debt corner solution, depicted in figure four by b_0, reduces the warrant value.

Figure 4: *The Constrained Maximization Problem* [10]



The 1958 Modigliani-Miller theorem suggests that the financing condition is irrelevant. Indeed, if the manager is paid with stock or with stock options that are safe[11] then the irrelevance results follows by direct calculation. If the manager is compensated with stock options that are not safe, as hypothesized here, then an irrelevance result need not follow. The following proposition shows that the manager is not indifferent to

[10]We have not been able to determine the curvature of the iso-warrant value lines. Indeed it may not be possible to say that they are convex or concave since the curvature depends on a pricing density. The results stated here, however, are distribution free.

[11]A stock option is safe if it can be exercised *then* with probability one.

the capital structure choices available. After exhausting the sources of internal financing, the manager prefers the debt choice to all possible mixes of debt and equity.

Proposition 3. Let $F(0) = 0$, $0 < F(b + e\,N) < 1$ and $0 < F(e\,(N + m)) < 1$ for $e > 0$. Then the manager prefers a debt issue to all possible combinations of debt and equity that finance the investment.

Proof. There is a corner solution to this problem if the iso-warrant value curves are steeper than the financing constraint. Recall that the warrant value is

$$W(b, e, m) = \frac{1}{1 + r} \int_{b+e(N+m)}^{\infty} \left(\frac{n}{N + m + n} \left(\pi_1 + e\,n - b\right) - e\,n \right) f(\pi_1)\, d\pi_1$$

It follows that the slope of the iso-warrant value line is

$$\frac{db}{dm} = -\frac{\frac{\partial W}{\partial m}}{\frac{\partial W}{\partial b}} = -\frac{\int_{b+e(N+m)}^{\infty} \frac{n}{\left(N + m + n\right)^2} \left(\pi_1 + e\,n - b\right) f(\pi_1)\, d\pi_1}{\int_{b+e(N+m)}^{\infty} \left(\frac{n}{N + m + n}\right) f(\pi_1)\, d\pi_1}$$

$$= -\frac{\int_{b+e(N+m)}^{\infty} \frac{1}{N + m + n} \left(\pi_1 + e\,n - b\right) f(\pi_1)\, d\pi_1}{\int_{b+e(N+m)}^{\infty} f(\pi_1)\, d\pi_1}$$

Similarly, recall that the financing constraint is

$$C(b, e, m) \equiv D(b) + p_s(b, e, m)\, m - \left(I_0 - \pi_0\right) = 0$$

$$\Leftrightarrow C(b, e, m) = \frac{1}{1+r}\int_0^b \pi_1\ f(\pi_1)\ d\pi_1 + \frac{1}{1+r}\int_b^\infty b\ f(\pi_1)\ d\pi_1$$

$$+ \frac{1}{1+r}\int_b^{b+e\left(N+m\right)} \frac{m}{N+m}\left(\pi_1 - b\right) f(\pi_1)\ d\pi_1$$

$$+ \frac{1}{1+r}\int_{b+e\left(N+m\right)}^\infty \frac{m}{N+m+n}\left(\pi_1 + e\,n - b\right) f(\pi_1)\ d\pi_1 - \left(I_0 - \pi_0\right) = 0$$

It follows that

$$\frac{db}{dm} = -\frac{\frac{\partial C}{\partial m}}{\frac{\partial C}{\partial b}}$$

where

$$\frac{\partial C}{\partial b} = \frac{1}{1+r}\int_b^\infty f(\pi_1)\ d\pi_1 - \frac{1}{1+r}\int_b^{b+e\left(N+m\right)} \frac{m}{N+m}\ f(\pi_1)\ d\pi_1$$

$$- \frac{1}{1+r}\int_{b+e\left(N+m\right)}^\infty \frac{m}{N+m+n}\ f(\pi_1)\ d\pi_1$$

and

$$\frac{\partial C}{\partial m} = \frac{1}{1+r}\int_{b}^{b+e(N+m)} \frac{N}{(N+m)^2}\left(\pi_1 - b\right) f(\pi_1)\, d\pi_1$$

$$+ \frac{1}{1+r}\int_{b+e(N+m)}^{\infty} \frac{N+n}{(N+m+n)^2}\left(\pi_1 + en - b\right) f(\pi_1)\, d\pi_1$$

Now, consider whether the following inequality holds.

$$\left.\frac{\dfrac{\partial W}{\partial m}}{\dfrac{\partial W}{\partial b}}\right|_{e>0} > \left.\frac{\dfrac{\partial H}{\partial m}}{\dfrac{\partial H}{\partial b}}\right|_{e>0} \qquad (27)$$

If it does then debt is preferred over all possible debt/equity mixes in financing the investment. Note that (27) is equivalent to[12]

$$\frac{\displaystyle\int_{b+e(N+m)}^{\infty} \frac{1}{N+m+n}\left(\pi_1 + en - b\right) f(\pi_1)\, d\pi_1}{\displaystyle\int_{b+e(N+m)}^{\infty} f(\pi_1)\, d\pi_1} >$$

$$\frac{\displaystyle\int_{b}^{b+e(N+m)} \frac{N}{(N+m)^2}\left(\pi_1 - b\right) f(\pi_1)\, d\pi_1 + \int_{b+e(N+m)}^{\infty} \frac{N+n}{(N+m+n)^2}\left(\pi_1 + en - b\right) f(\pi_1)\, d\pi_1}{\displaystyle\int_{b}^{\infty} f(\pi_1)\, d\pi_1 - \int_{b}^{b+e(N+m)} \frac{m}{N+m} f(\pi_1)\, d\pi_1 - \int_{b+e(N+m)}^{\infty} \frac{m}{N+m+n} f(\pi_1)\, d\pi_1}$$

[12]The intermediate steps, used in establishing the following sequence of inequalities, are provided in the appendix.

$$\Leftrightarrow \frac{F\left(b+e\left(N+m\right)\right) - F(b)}{1 - F\left(b+e\left(N+m\right)\right)} \int_{b+e\left(N+m\right)}^{\infty} \frac{1}{N+m+n} \left(\pi_1 + en - b\right) f(\pi_1)\, d\pi_1 >$$

(28)

$$\int_{b}^{b+e\left(N+m\right)} \frac{1}{N+m} \left(\pi_1 - b\right) f(\pi_1)\, d\pi_1$$

Note that (28) holds since[13]

$$\frac{F\left(b+e\left(N+m\right)\right) - F(b)}{1 - F\left(b+e\left(N+m\right)\right)} \int_{b+e\left(N+m\right)}^{\infty} \frac{1}{N+m+n} \left(\pi_1 + en - b\right) f(\pi_1)\, d\pi_1 >$$

$$\frac{F\left(b+e\left(N+m\right)\right) - F(b)}{1 - F\left(b+e\left(N+m\right)\right)}\, e\left[1 - F\left(b+e\left(N+m\right)\right)\right] =$$

$$e\left[F\left(b+e\left(N+m\right)\right) - F(b)\right] >$$

$$\int_{b}^{b+e\left(N+m\right)} \frac{1}{N+m} \left(\pi_1 - b\right) f(\pi_1)\, d\pi_1$$

Q.E.D.

4. Concluding Remarks

The existing literature on the pecking order theory is based on the assumption that agents are asymmetrically informed. The asymmetry assumption led to a debate in

[13]To see the following, evaluate the integral on the LHS at its lower limit and the integral on the RHS at its upper limit.

the literature about whether it was possible to reveal information to market agents through the capital structure choice and whether revelation of the information was or was not costly. This analysis eliminates that conundrum by supposing that market agents are symmetrically informed. Although asymmetric information was thought to be a key reason in explaining the pecking order theory, this analysis shows that the compensation contract plays a pivotal role in the manager's capital structure decisions.

The stock option contract provides the manager with an incentive to make capital structure decisions that do not dilute the manager's stake in the corporation. This incentive is manifested in the preference for internal equity, equivalently, retained earnings, over debt and outside equity in financing an investment. This is a stronger result than the Myers and Majluf result because there the manager was indifferent between using retained earnings versus safe debt. The incentive is also manifested in a preference for debt over outside equity. This result is also stronger than the Myers and Majluf result because their result showed a preference for safe debt over equity while the results here show that even risky debt is preferred to equity. What is more, the results here show that the manager prefers debt to any combinations of debt and equity. The results here do not require the risk neutrality arguments so often made in literature, nor do they require the assumption that financial markets are complete. Therefore the results are robust and provide the basis for a more powerful test of the pecking order hypothesis.

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Notes on Proposition 2

To establish the inequality in (26), note that the difference in warrant values may be expressed as follows:

$$W(b, e, 0) - W(0, e, m(e)) = \frac{1}{1+r} \int_{b+eN}^{\infty} \left(\delta\, \pi_1 - \left(1-\delta\right) e\, n - \delta\, b\right) f(\pi_1)\, d\pi_1$$

$$- \frac{1}{1+r} \int_{e\left(N+m(e)\right)}^{\infty} \left(\xi(e)\, \pi_1 - \left(1-\xi(e)\right) e\, n\right) f(\pi_1)\, d\pi_1$$

$$= \frac{1}{1+r} \int_{b+eN}^{\infty} \left[\left(\delta\, \pi_1 - \left(1-\delta\right) e\, n - \delta\, b\right) - \left(\xi(e)\, \pi_1 - \left(1-\xi(e)\right) e\, n\right)\right] f(\pi_1)\, d\pi_1$$

$$- \frac{1}{1+r} \int_{e\left(N+m(e)\right)}^{b+eN} \left(\xi(e)\, \pi_1 - \left(1-\xi(e)\right) e\, n\right) f(\pi_1)\, d\pi_1 \qquad \text{(A.1)}$$

Then

$$\frac{\partial}{\partial e}\left(W(b, e, 0) - W(0, e, m(e))\right)$$

$$= \frac{1}{1+r} \int_{b+eN}^{\infty} \left[-\left(1-\delta\right)n + \left(1-\xi(e)\right)n - \left(\xi'(e)\, \pi_1 + \xi'(e)\, e\, n\right)\right] f(\pi_1)\, d\pi_1$$

$$- \frac{1}{1+r} \int_{e\left(N+m(e)\right)}^{b+eN} \left(\xi'(e)\, \pi_1 + \xi'(e)\, e\, n - \left(1-\xi(e)\right)n\right) f(\pi_1)\, d\pi_1$$

$$= \frac{1}{1+r} \int_{b+eN}^{\infty} \left[\left(\delta - \xi(e)\right) n - \xi'(e) \left(\pi_1 + e\ n\right) \right] f(\pi_1)\ d\pi_1$$

$$- \frac{1}{1+r} \int_{e\left(N+m(e)\right)}^{b+eN} \left(\xi'(e) \left(\pi_1 + e\ n\right) - \left(1 - \xi(e)\right) n \right) f(\pi_1)\ d\pi_1$$

$$= \frac{1}{1+r} \int_{b+eN}^{\infty} \left(\delta - \xi(e)\right) n\ f(\pi_1)\ d\pi_1 + \frac{1}{1+r} \int_{e\left(N+m(e)\right)}^{b+eN} \left(1 - \xi(e)\right) n\ f(\pi_1)\ d\pi_1$$

$$- \frac{1}{1+r} \xi'(e) \int_{e\left(N+m(e)\right)}^{\infty} \left(\pi_1 + e\ n\right) f(\pi_1)\ d\pi_1 \qquad \text{(A.2)}$$

Next, consider how ξ changes with respect to the exercise price. Note that

$$\xi' = - \frac{n\ m'}{\left(N + n + m\right)^2} = \xi \frac{-\ m'}{N + n + m}$$

and so the sign of ξ' is the opposite of the sign of m'. The number of new shares that must be issued is implicitly defined by the financing condition in (20) or equivalently by

$$\frac{1}{1+r} \left(\int_{0}^{e\left(N+m\right)} \frac{m}{N+m} \pi_1\ f(\pi_1)\ d\pi_1 + \int_{e\left(N+m\right)}^{\infty} \frac{m}{N+m+n} \left(\pi_1 + e n\right) f(\pi_1)\ d\pi_1 \right)$$

$$- \left(I_0 - \pi_0\right) = 0 \qquad \text{(A.3)}$$

The first two terms on the LHS of (A.3) represent the value of the new shares, i.e., p(0, e, m(e)) m(e). Then

$$m' = -\frac{\frac{\partial}{\partial e}\left(p(0, e, m)\ m\right)}{\frac{\partial}{\partial m}\left(p(0, e, m)\ m\right)}$$

$$= -\frac{\int_{e\left(N+m\right)}^{\infty} \frac{m\ n}{N + m + n}\ f(\pi_1)\ d\pi_1}{\frac{N}{\left(N + m\right)^2} \int_0^{e\left(N+m\right)} \pi_1\ f(\pi_1)\ d\pi_1 + \frac{N + n}{\left(N + m + n\right)^2} \int_{e\left(N+m\right)}^{\infty} \left(\pi_1 + e\,n\right) f(\pi_1)\ d\pi_1} \quad \text{(A.4)}$$

Now rewrite (A.2) to get

$$= \frac{1}{1 + r} \int_{b+eN}^{\infty} \left(\delta - \xi(e)\right) n\ f(\pi_1)\ d\pi_1 + \frac{1}{1 + r} \int_{e\left(N+m(e)\right)}^{b+eN} \left(1 - \xi(e)\right)\ n\ f(\pi_1)\ d\pi_1$$

$$- \frac{1}{1 + r}\ \xi\ \frac{-\ m'}{N + n + m} \int_{e\left(N+m(e)\right)}^{\infty} \left(\pi_1 + e\ n\right) f(\pi_1)\ d\pi_1$$

Using (A.4) yields

$$= \frac{1}{1 + r} \int_{b+eN}^{\infty} \left(\delta - \xi(e)\right) n\ f(\pi_1)\ d\pi_1 + \frac{1}{1 + r} \int_{e\left(N+m(e)\right)}^{b+eN} \left(1 - \xi(e)\right)\ n\ f(\pi_1)\ d\pi_1$$

$$- \frac{1}{1 + r} \left[\frac{\int_{e\left(N+m\right)}^{\infty} \frac{m\ n}{N + m + n}\ f(\pi_1)\ d\pi_1}{\frac{N}{\left(N + m\right)^2} \int_0^{e\left(N+m\right)} \pi_1\ f(\pi_1)\ d\pi_1 + \frac{N + n}{\left(N + m + n\right)^2} \int_{e\left(N+m\right)}^{\infty} \left(\pi_1 + e\,n\right) f(\pi_1)\ d\pi_1} \right]$$

$$\times \int_{e\left(N+m(e)\right)}^{\infty} \frac{n\ (\pi_1 + e\ n)}{(N + n + m)^2}\ f(\pi_1)\ d\pi_1$$

$$> \frac{1}{1+r}\int_{b+eN}^{\infty}\left(\delta-\xi(e)\right)n\ f(\pi_1)\ d\pi_1+\frac{1}{1+r}\int_{e\left(N+m(e)\right)}^{b+eN}\left(1-\xi(e)\right)\ n\ f(\pi_1)\ d\pi_1$$

$$-\frac{1}{1+r}\int_{e\left(N+m(e)\right)}^{\infty}\frac{n}{N+n}\ \frac{n}{N+n+m(e)}\ m(e)\ f(\pi_1)\ d\pi_1 \tag{A.5}$$

The inequality in (A.5) holds if and only if

$$\frac{\frac{N+n}{n}\frac{n}{(N+n+m)^2}\int_{e\left(N+m(e)\right)}^{\infty}\left(\pi_1+e\ n\right)f(\pi_1)\ d\pi_1}{\frac{N}{\left(N+m\right)^2}\int_0^{e\left(N+m\right)}\pi_1\ f(\pi_1)\ d\pi_1+\frac{N+n}{\left(N+m+n\right)^2}\int_{e\left(N+m\right)}^{\infty}\left(\pi_1+en\right)f(\pi_1)\ d\pi_1}<1$$

$$\Leftrightarrow \frac{N+n}{(N+n+m)^2}\int_{e\left(N+m(e)\right)}^{\infty}\left(\pi_1+e\ n\right)f(\pi_1)\ d\pi_1<$$

$$\frac{N}{\left(N+m\right)^2}\int_0^{e\left(N+m\right)}\pi_1\ f(\pi_1)\ d\pi_1+\frac{N+n}{\left(N+m+n\right)^2}\int_{e\left(N+m\right)}^{\infty}\left(\pi_1+en\right)f(\pi_1)\ d\pi_1$$

$$\Leftrightarrow 0<\frac{N}{\left(N+m\right)^2}\int_0^{e\left(N+m\right)}\pi_1\ f(\pi_1)\ d\pi_1$$

Hence, the inequality in (A.5) holds because $F(0) = 0$ and $F(e\ (N + m)) > 0$.

Next, the RHS of (A.5) may be equivalently expressed as follows

$$\frac{1}{1+r}\int_{b+eN}^{\infty}\left(\delta-\xi(e)\right)n\ f(\pi_1)\ d\pi_1+\frac{1}{1+r}\int_{e\left(N+m(e)\right)}^{b+eN}\left(1-\xi(e)\right)\ n\ f(\pi_1)\ d\pi_1$$

$$-\frac{1}{1+r}\int_{e\left(N+m(e)\right)}^{\infty}\frac{n}{N+n}\ \frac{n}{N+n+m(e)}\ m(e)\ f(\pi_1)\ d\pi_1$$

$$= \frac{1}{1+r}\int_{b+eN}^{\infty}\left(\delta-\xi(e)\right)n\ f(\pi_1)\ d\pi_1 + \frac{1}{1+r}\int_{e\left(N+m(e)\right)}^{b+eN}\left(1-\xi(e)\right)\ n\ f(\pi_1)\ d\pi_1$$

$$-\frac{1}{1+r}\int_{e\left(N+m(e)\right)}^{\infty}\left(\delta\ \xi(e)\ m(e)\right)\ f(\pi_1)\ d\pi_1$$

$$= \frac{1}{1+r}\int_{b+eN}^{\infty}\left[\left(\delta-\xi(e)\right)n-\delta\ \xi(e)\ m(e)\right]\ f(\pi_1)\ d\pi_1 + \frac{1}{1+r}\int_{e\left(N+m(e)\right)}^{b+eN}\left(1-\xi(e)\right)\ n\ f(\pi_1)\ d\pi_1$$

$$-\frac{1}{1+r}\int_{e\left(N+m(e)\right)}^{b+eN}\delta\ \xi(e)\ m(e)\ f(\pi_1)\ d\pi_1$$

$$= \frac{1}{1+r}\int_{e\left(N+m(e)\right)}^{b+eN}\left[\left(1-\xi(e)\right)\ n-\delta\ \xi(e)\ m(e)\right]\ f(\pi_1)\ d\pi_1$$

$$= \frac{1}{1+r}\int_{e\left(N+m(e)\right)}^{b+eN}\frac{N}{N+n}\ n\ f(\pi_1)\ d\pi_1{}^{14} \geq 0 \qquad \text{(A.6)}$$

(A.6) establishes the inequality in (26).

Notes on Proposition 3

The intermediate steps in establishing the inequality in (28) are as follows:

$$\frac{\int_{b+e\left(N+m\right)}^{\infty}\frac{1}{N+m+n}\left(\pi_1+en-b\right)\ f(\pi_1)\ d\pi_1}{\int_{b+e\left(N+m\right)}^{\infty}f(\pi_1)\ d\pi_1} >$$

[14] It follows by direct calculation that (δ - ξ(e)) n - δ ξ(e) m(e) = 0 and (1 -ξ(e)) n - δ ξ(e) m(e) = N n/(N + n).

$$\frac{\int_b^{b+e(N+m)} \frac{N}{(N+m)^2}(\pi_1 - b) f(\pi_1)\, d\pi_1 + \int_{b+e(N+m)}^{\infty} \frac{N+n}{(N+m+n)^2}(\pi_1 + en - b) f(\pi_1)\, d\pi_1}{\int_b^{\infty} f(\pi_1)\, d\pi_1 - \int_b^{b+e(N+m)} \frac{m}{N+m} f(\pi_1)\, d\pi_1 - \int_{b+e(N+m)}^{\infty} \frac{m}{N+m+n} f(\pi_1)\, d\pi_1}$$

$$\Leftrightarrow \frac{\int_{b+e(N+m)}^{\infty} \frac{1}{N+m+n}(\pi_1 + en - b) f(\pi_1)\, d\pi_1}{\int_{b+e(N+m)}^{\infty} f(\pi_1)\, d\pi_1} >$$

$$\frac{\int_b^{b+e(N+m)} \frac{N}{(N+m)^2}(\pi_1 - b) f(\pi_1)\, d\pi_1 + \int_{b+e(N+m)}^{\infty} \frac{N+n}{(N+m+n)^2}(\pi_1 + en - b) f(\pi_1)\, d\pi_1}{\int_b^{b+e(N+m)} \frac{N}{N+m} f(\pi_1)\, d\pi_1 + \int_{b+e(N+m)}^{\infty} \frac{N+n}{N+m+n} f(\pi_1)\, d\pi_1}$$

$$\Leftrightarrow \frac{\int_b^{b+e(N+m)} \frac{N}{N+m} f(\pi_1)\, d\pi_1 + \int_{b+e(N+m)}^{\infty} \frac{N+n}{N+m+n} f(\pi_1)\, d\pi_1}{\int_{b+e(N+m)}^{\infty} f(\pi_1)\, d\pi_1}$$

$$\times \int_{b+e(N+m)}^{\infty} \frac{1}{N+m+n}(\pi_1 + en - b) f(\pi_1)\, d\pi_1 >$$

$$\int_b^{b+e(N+m)} \frac{N}{(N+m)^2}(\pi_1 - b) f(\pi_1)\, d\pi_1 + \int_{b+e(N+m)}^{\infty} \frac{N+n}{(N+m+n)^2}(\pi_1 + en - b) f(\pi_1)\, d\pi_1$$

$$\Leftrightarrow \left(\frac{N}{N+m} \frac{\int_b^{b+e(N+m)} f(\pi_1)\, d\pi_1}{\int_{b+e(N+m)}^{\infty} f(\pi_1)\, d\pi_1} + \frac{N+n}{N+m+n} \right) \int_{b+e(N+m)}^{\infty} \frac{1}{N+m+n}(\pi_1 + en - b) f(\pi_1)\, d\pi_1 >$$

$$\int_b^{b+e(N+m)} \frac{N}{(N+m)^2} (\pi_1 - b)\, f(\pi_1)\, d\pi_1 + \int_{b+e(N+m)}^{\infty} \frac{N+n}{(N+m+n)^2} (\pi_1 + en - b)\, f(\pi_1)\, d\pi_1$$

$$\Leftrightarrow \frac{N}{N+m} \frac{\int_b^{b+e(N+m)} f(\pi_1)\, d\pi_1}{\int_{b+e(N+m)}^{\infty} f(\pi_1)\, d\pi_1} \int_{b+e(N+m)}^{\infty} \frac{1}{N+m+n} (\pi_1 + en - b)\, f(\pi_1)\, d\pi_1 >$$

$$\int_b^{b+e(N+m)} \frac{N}{(N+m)^2} (\pi_1 - b)\, f(\pi_1)\, d\pi_1$$

$$\Leftrightarrow \frac{F(b+e(N+m)) - F(b)}{1 - F(b+e(N+m))} \int_{b+e(N+m)}^{\infty} \frac{1}{N+m+n} (\pi_1 + en - b)\, f(\pi_1)\, d\pi_1 >$$

$$\int_b^{b+e(N+m)} \frac{1}{N+m} (\pi_1 - b)\, f(\pi_1)\, d\pi_1$$