Stock Options and the Corporate Demand for Insurance

by

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Most of the existing literature on the corporate demand for insurance rests either implicitly or explicitly on the notion that the decisions on corporate account are made to maximize the current shareholder value. (Mayers and Smith 1982) began a discussion of the determinants of the demand for corporate insurance by noting that the ". . . corporate form provides and effective hedge since stockholders can eliminate insurable risk through diversification." Equivalently, the value of the insured firm is equal to that of the uninsured firm and insurance plays no role in the management of corporate risk.[1] The role insurance plays in managing corporate risk has since been clarified by (Main 1983; MacMinn 1987; Mayers and Smith 1987; MacMinn and Han 1990; Garven and MacMinn 1993; Han 1996). (MacMinn 1987) shows that the risk-shifting problem can be solved with an insurance contract and so increase the value of the corporation;[2] similarly (Mayers and Smith 1987; Garven and MacMinn 1993) show that the under-investment problem can be solved with insurance.[3] These results were derived assuming the maximization of current shareholder value. Stock option grants, however, have become an increasingly important component of executive compensation in the last two decades of the twentieth century (Murphy 1998; Murphy 1999). Stock options are supposed to align the incentives of management and shareholder since the options give management the incentive to increase the share price. The deductive foundation for this conventional wisdom has not been provided in the literature. Hence, the objective here is first to provide the link between the executive compensation scheme and corporate decision making and second to examine the impact of the executive compensation on the corporate demand for insurance and third to examine the role insurance plays in aligning the interest of executives and shareholders.

The literature on the demand for corporate insurance is one thread of the broader literature on risk management. In the risk management literature (Smith and Stulz 1985) consider a managerial motive that provides a linkage between compensation and corporate decision making. Smith and Stulz show that the risk averse manager compensated with stock will use forward contracts to hedge risk; they also show that when the compensation is stock options, the options will ultimately eliminate the incentive to hedge.[4] There is some empirical support for the managerial theory in (Tufano 1996).[5] The Smith and Stulz model differs from that here because they do not allow the corporate executive to hold a portfolio on personal account or diversify that portfolio. The managerial analysis is reframed here and the corporate objective function is explicitly derived for the manager paid in stock options; then the analysis shows that the stock options eliminate the incentive to hedge with forward contracts.[6] This might suggest that neither will the manager use insurance to manage risk but this model shows that not all risk management tools are created equal. The forward

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[1] This is a corollary of the 1958 Modigliani-Miller theorem which shows that the value of the levered firm equals that of the unlevered firm and implies that capital structure decisions are irrelevant.

[2] The risk-shifting problem was originally solved by Green, R. C. (1984). "Investment Incentives, Debt and Warrants." Journal of Financial Economics **13**: 115-36. See MacMinn, R. D. (1993). On The Risk Shifting Problem and Convertible Bonds. Advances in Quantitative Analysis of Finance and Accounting. for an alternative proof.

[3] These articles are apparently the first demonstration of a solution to the under-investment problem.

[4] Also see Carpenter, J. N. (2000). "Does Option Compensation Increase Managerial Risk Appetite?" Journal of Finance **55**(5): 2311-2331. for the effects of a convex compensation scheme on the behavior of a risk averse manager.

[5] Tufano studies the risk management practices in the gold mining industry and finds that managers who own more stock options manage gold price risk less using forward sales, gold loans, options, and other hedging activities as measures of risk management.

[6] The objective function could also be derived for the manager paid in stock to show that such a manager would not have the incentive to use forward contracts.

contract reduces risk by eliminating weight from the tails of the corporate earnings distribution and this reduces the value of the stock options. The liability insurance considered here requires a premium *now* for coverage *then*; it transfers of cash from out-of-the-money states to in-the-money states, increases the value of the stock options and alters the incentives of the manager.

There is a large and growing empirical literature on executive compensation and corporate performance. (DeFusco, Johnson et al. 1990) provide an early example that the use of stock options is consistent with increasing corporate risk. Main (Main 1991) studied a sample of Britain's largest industrial firms in an attempt to estimate the link between executive pay and shareholder wealth; he found some evidence that executive compensation policy aligns executive and shareholder interests. Lewellen *et. al.* (Lewellen 1992) studied a sample of large industrial firms to determine the relationship between top executive compensation and firm performance; they find a positive relation and conclude that executive compensation is designed to reduce agency costs. Mayers and Smith (Mayers and Smith 1992) considered differences between compensation in mutual versus stock insurance companies; they found the compensation in mutual insurance companies to be less responsive to firm performance than the stock insurance companies and found that result consistent with the difference in investment opportunity sets between the two firm types. Some of the more recent studies have tested the management entrenchment hypothesis by examining firms that have reset the option exercise prices (Acharya 1992; Acharya, John et al. 2000; Bebchuk, Fried *et al.* 2002; Chidambaran and Prabhala 2003); the evidence is mixed.

While it may be too early to expect definitive empirical results on the incentive effects of stock options,[7] it is possible to provide a theoretical foundation for the effects. In the next section, a financial market model is constructed which allows the linkage between executive compensation and corporate performance to be established. The standard economic assumption of self-interested behavior is made here and the analysis shows that if a risk- averse corporate manager is paid in stock options then the manager makes decisions on corporate account to maximize the value of the stock option package, i.e., independent of preferences for consumption *now* versus *then*. In this model the manager makes investment and financing decisions on behalf of the corporation. The analysis first shows that the corporate manager over-invests relative to the investment level that maximizes the current shareholder value. The analysis shows that the stock options fail to align the manager's interests with those of the shareholders; equivalently, the use of stock options in the compensation package creates an agency problem for the corporation by separating the interest of management and shareholders. This analysis also shows that the manager paid in stock options has no incentive to hedge with forward contracts. In the penultimate section, insurance is introduced and management makes an insurance decision as well as the investment and financing decisions. The analysis shows when insurance allows for a transfer of cash from the out-of-the-money states to the in-the-money states the manager has an incentive to purchase insurance. Finally, the analysis shows that the insurance allows a better alignment between the interests of shareholders and management. The final section provides concluding remarks.

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[7] It should be noted that Tufano has provided empirical support for the hedging theorem provided here in addition to the support he claims for the Smith and Stulz managerial theory. It would be useful to find a test that can distinguish between the two models.

The Uninsured Corporation

To introduce the first version of the model, consider a competitive economy operating between the dates *now* and *then*. The economy is composed of many individuals and firms. All choices are made *now* and the uncertain consequences of those choices are experienced *then*. The risk averse individuals make choices for consumption *now* and *then*; in a financial market setting this means that consumers select a portfolio of financial assets, i.e., saving, that determines consumption *now* and *then*. Firms make investment and financing choices that payoff *then*. The financial markets are assumed to be competitive.[8] The analysis will focus on the behavior of a firm which is controlled by the decisions of its risk averse chief executive officer (CEO); the standard assumption that agents act in their own self-interests is employed here for the firm. The CEO is faced with two sets of decisions. The first set consists of the portfolio decisions on personal account and the second is the set consists of the investment and financing decisions made on corporate account. The analysis here will show that a Fisher separation result holds; acting in the pursuit of self-interests yields a well defined objective function for the firm which does not depend on inter-temporal consumption preferences or on risk aversion. The analysis will also show how the CEO's compensation scheme influences the corporate decisions.

Stock options have become an important component of executive compensation schemes and are used here to motivate the CEO. The conventional wisdom has been that the stock options provide the manger with an incentive to increase the corporate share price and so align the interests of management with that of shareholders. To capture the pure stock option effects, we will suppose that the CEO is paid with stock options alone.[9] The manager is granted the stock options *now* and the options vest *then*. This framework allows the conventional wisdom to be tested. The CEO then makes all decisions on personal and corporate account to maximize expected utility subject to a budget constraint on personal account and a financing constraint on corporate account. The budget constraint holds the value of the consumption plan to the risk adjusted present value of the income stream. The financial constraint specifies that the value of the debt issue covers the investment expenditure; the financial constraint could include a new stock issue as well as a debt issue but MacMinn and Page have shown that a pecking order results if the manager is paid in stock options (MacMinn and Page 1995) and so the debt issue is assumed here. The following notations are used in the model here:

ω	state of nature
$\Omega = (0, \zeta)$	set of states of nature
$\Psi(\omega)$	Probability distribution for the states

[8] To make the uncertainty operational suppose that the consumer can transfer dollars from *now* to *then* by purchasing one or more risky assets; each basis asset in a complete market model is a promise to pay one dollar if state of nature ω occurs and zero otherwise. Let $p(\omega)$ be the price of an asset that yields one dollar in state ω and zero otherwise.

[9] A fixed salary could also be added without affecting any of the results. Including bonuses or stock (restricted or not) would alter the results if given sufficient weight in the compensation scheme and deserve a separate focus.

$\Pi(I, \omega)$	payoff function for the corporation where $\frac{\partial \Pi}{\partial I} > 0, \frac{\partial^2 \Pi}{\partial^2 I} < 0$
$L(I, \omega)$	Loss function for the corporation where $\frac{\partial L}{\partial I} > 0$
$(c_{i0}, c_{i1}(\omega))$	consumption *now* and *then* in state ω by individual i
$(m_{i0}, m_{i1}(\omega))$	income *now* and in state ω by individual i
$p(\omega)$	basis stock price
$P(\omega)$	sum of the basis stock prices up to ω, i.e., $P(\omega) = \int_0^\omega dP(t)$
p	Corporate share price
m	number of stock options
N	number of shares of common stock *now*
b	Promised repayment on a zero coupon bond issued to finance the corporate investment and insurance
a	manager's stake given exercise; $a \equiv \frac{m}{N+m}$
e	stock option exercise price *then*
W^j	warrant value, j = u, i, where u and i correspond to the uninsured and insured cases, respectively
V^j	corporate value, j = u, i

In this financial market setting the payoff on the firm's zero coupon bond issue is $\max\left\{0, \min\left\{\Pi\left(I, \omega\right) - L\left(I, \omega\right), b\right\}\right\}$ and so value of the firm's debt issue is D(I, b) where

$$D(I, b) = \int_0^{\zeta} \max\left\{0, \min\left\{\Pi(I, \omega) - L(I, \omega), b\right\}\right\} dP(\omega)$$
$$= \int_{\gamma}^{\zeta} \min\left\{\Pi(I, \omega) - L(I, \omega), b\right\} dP(\omega) \quad (1)$$

where γ is the boundary of the insolvency event or equivalently the state at which the net corporate payoff equals the promised debt payment. Suppose the manager is paid in stock options *now*. Let each option give the manager the right to purchase one share of corporate stock *then* at an exercise price of e dollars. Suppose the manager is paid with m options *now*. The corporation has N shares outstanding *now* and issues an additional m shares *then* if the manager exercises the options. Without loss of generality, suppose the firm issues bonds *now* to cover its investment expenditure. The manager selects a consumption pair, equivalently portfolio on personal account and an investment level on corporate account to solve the following constrained maximization problem[10]

$$\text{maximize} \int_{\Omega} u_i\left(c_{i0}, c_{i1}(\omega)\right) d\Psi_i(\omega)$$

$$\text{subject to}: c_{i0} + \int_{\Omega} c_{i1}(\omega)\, dP(\omega) = m_{i0} + \int_{\Omega} m_{i1}(\omega)\, dP(\omega)$$
$$+ \int_{\Omega} \max\left\{0, \frac{m}{N + m}\left(\Pi(I, \omega) - L(I, \omega) + e\, m - b\right) - e\, m\right\} dP(\omega) \quad (2)$$

$$\text{and} \int_{\Omega} \max\left\{0, \min\left\{\Pi(I, \omega) - L(I, \omega), b\right\}\right\} dP(\omega) = I$$

The last expression in the problem is the financing constraint; it determines the promised payment b on the debt issued *now* to raise I dollars for investment. The problem may also be expressed in reduced form by noting that the financing constraint implicitly defines a function b(I).[11] Substituting this function into the budget constraint and simplifying yields the constrained maximization problem in the following reduced form:

$$\text{maximize} \int_{\Omega} u_i\left(c_{i0}, c_{i1}(\omega)\right) d\Psi_i(\omega)$$
$$\text{subject to } c_{i0} + \int_{\Omega} c_{i1}(\omega)\, dP(\omega) = m_{i0} + \int_{\Omega} m_{i1}(\omega)\, dP(\omega) + W(I) \quad (3)$$

where W represents the value of the stock option package, or equivalently, the warrant value.[12] The warrant value may be equivalently expressed as

[10] This constrained maximization problem can be expressed in several forms and may include investments in other corporate assets without altering the conclusions reached here.
[11] See lemma one for a derivation of this function in the more general case.
[12] A warrant usually refers to an option that is issued by a corporation and so becomes part of its capital structure whereas a call option trades against the same underlying asset but is not issued by the corporation

$$W(I) = \int_{\Omega} \max\left\{0, \frac{m}{N + m}\left(\Pi(I, \omega) - L(I, \omega) + e\ m - b\right) - e\ m\right\} dP(\omega)$$
$$= \int_{\Omega} \max\left\{0, a\left(\Pi(I, \omega) - L\left(I, \omega\right) - b\right) - (1 - a)\ E\right\} dP \tag{4}$$

where E = e m is the gross exercise value. In this form it is clear that the manager makes decisions on corporate account to maximize the warrant value. Hence, we have a **Fisher Separation** result for the **publicly held and traded corporation**. This result specifies the corporate objective function and is noted in the next proposition.

Fisher Separation Theorem.[13] The manager paid in stock options *now* that vest *then* makes decisions on corporate account that are independent of the manager's preferences for consumption now versus then; the decisions are made to maximize the warrant value W(I).

There are several versions of Fisher separation. Each depends on how the corporate manager is compensated. If the manager is paid in shares of corporate stock then the same method, used in the appendix to prove this theorem, shows that the manager makes decisions on corporate account to maximize the stock value.

The manager makes the investment decision on corporate account to maximize the value of the objective function determined by the compensation scheme. The warrant scheme puts the payoff in the right tail of the distribution and motivates the manager to increase the probability weight there. How that motivation affects the investment decision depends on how the scale of the investment affects the payoff distribution. We will suppose in what follows that the Principle of Increasing Uncertainty (PIU) (Leland 1972) applies to the net payoff $\Pi - L$ so that the following derivative properties hold:

$$\frac{\partial\left(\Pi - L\right)}{\partial I} > 0; \ \frac{\partial^2\left(\Pi - L\right)}{\partial\omega\ \partial I} > 0 \tag{5}$$

The PIU yields an increase in risk in the Rothschild-Stiglitz sense after correcting for a change in the mean of the payoff distribution (MacMinn and Holtmann 1983); equivalently, increasing the scale of the investment will increase the weight in the tails of the payoff distribution.

The next theorem compares the optimal investment choices of the manager paid in stock options versus stock. The I^W and I^S denote the optimal choices of the managers paid in stock options and stock, respectively.

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and is not part of its capital structure. See Merton, R. C. (1973). "Theory of Rational Option Pricing." Bell Journal of Economics and Management Science **4**(1): 141-83.

[13] The proof of this version of the Fisher separation result is provided in the appendix. It as all other versions flows from the more is preferred to less assumption.

Theorem 1. The manager compensated with stock options selects a larger investment than that same manager compensated with an equivalent equity stake, $I^W > I^S$.

Proof. The stock value in the absence of stock options is

$$S(I) = \int_{\Omega} \max\{0,\ \Pi(I,\omega) - L(I,\omega) - b(I)\}\ dP(\omega)$$
$$= \int_0^{\zeta} \big(\Pi(I,\omega) - L(I,\omega) - b(I)\big)\ dP(\omega) \tag{6}$$

where b(I) is implicitly defined by the financing condition D(b) = I. The manager paid in stock selects the investment level to maximize S(I). The manager paid in stock options selects the investment level to maximize W(I) and that investment level is larger if

$$\frac{\partial W}{\partial I} - \frac{\partial S}{\partial I} > 0 \tag{7}$$

when evaluated at I^S. Letting β denote the boundary of the exercise event as shown in figure one, note that

$$\left.\frac{\partial W}{\partial I} - \frac{\partial S}{\partial I}\right|_{I=I^S} = \int_{\beta}^{\zeta}\left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b}{\partial I}\right)dP - \int_0^{\zeta}\left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b}{\partial I}\right)dP$$
$$= -\int_0^{\beta}\left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b}{\partial I}\right)dP \tag{8}$$
$$> 0$$

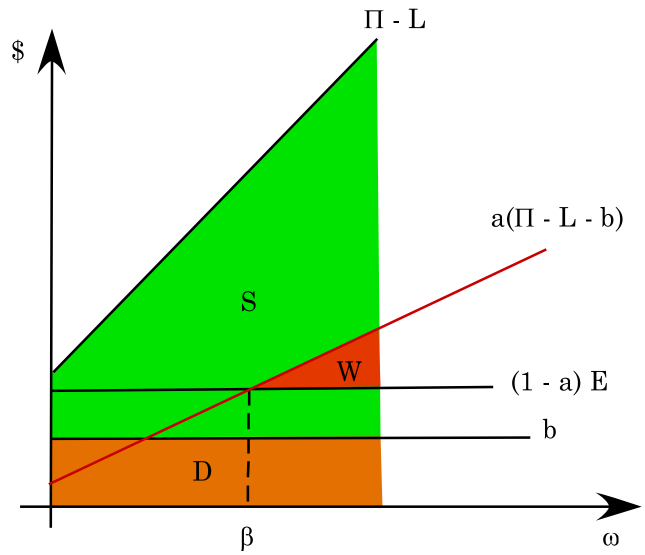
Since $\partial b/\partial I$ is a constant, the PIU suffices to show that

$$\frac{\partial}{\partial \omega}\left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b}{\partial I}\right) > 0$$

and so $\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b}{\partial I}$ goes from negative to positive monotonically. Hence (8) is positive when evaluated at $I = I^S$. Since (8) is positive when evaluated at I^S, it follows that $I^W > I^S$. QED

Theorem one establishes the benchmark. The CEO makes decisions to maximize the value of the warrants and in doing so selects an investment level that does not maximize the stock value of the corporation; the manager selects a larger investment that puts more weight in the tails of the payoff distribution and increases the risk. The risk is pushed too far from the shareholder perspective but the manager does not share in the downside risk with the shareholders. The division of value is represented in figure one. The risk adjusted values of the areas depicted by S, W and D represent the stock, warrant and debt values, respectively, of the corporation.

Figure 1: Values



An insurance contract is one of a number of contracts that the firm can use in managing corporate risk. It will be considered in the next section. It is, however, also possible to use forward and futures contracts to manage the same risk. To the extent that both insurance and forward contracts can be used to hedge risk, it is natural to claim that the two contracts are substitutes. If they are perfect substitutes then it is only necessary to use one; similarly, if the firm chooses not to use one then it will not use the other. Smith and Stulz (Smith and Stulz 1985) show that the manager compensated with stock options will have less incentive to hedge with forward contracts.

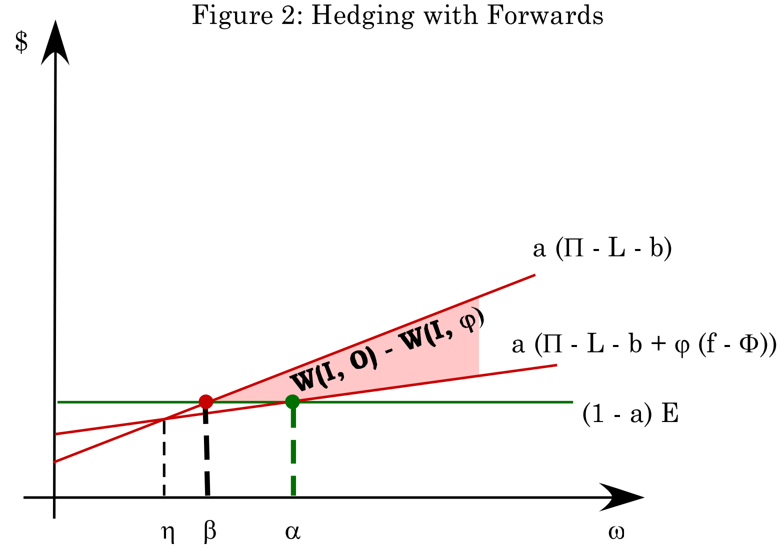
> "... if the manager's end-of-period utility is a convex function of the end-of-period firm value, Jensen's Inequality implies that the manager's end-of-period utility has a higher expected value if the firm is not hedged at all. Bonus or stock option provisions of compensation plans can make the manager's expected utility a convex function of the value of the firm. If the manager's expected utility is a convex function of the value of the firm, the manager will behave like a risk-seeker even though his expected utility function is a concave function of his end-of-period wealth."

Smith and Stulz consider a risk averse manager with no ability to diversify on personal account. The model here provides for the diversification on personal account and shows that it yields a corporate objective function which eliminates risk aversion in corporate decision making. The distinction is significant because the manager in the Smith and Stulz model who is paid in stock has the incentive to hedge risk but that is due to the risk aversion; the motivation is altered by stock options only because sufficient options make the objective function used by the manager convex rather than concave.[14] The distinction is also important because the manager paid in stock in this model has no incentive to hedge and that motivation is not qualitatively altered by stock options. Since such a result on stock options and hedging does not exist in the literature, it is provided in the next theorem.

[14] The objective function is the expected value of the composition of functions; one is the utility function and the other is the compensation which is a function of the hedge. If the compensation is stock then the second function is linear and the composite function is concave. If the compensation is stock options then the second function is convex and the composite function may be convex.

Suppose the financial market includes a stock index. Let Φ denote the unit payoff on the stock index and suppose the payoff increases in state. Further, suppose that forward contracts are traded with a unit gain or loss *then* of (f - Φ(ω)) where f is the forward price. The corporate payoff becomes $\Pi - L + \varphi(f - \Phi) - b$ as shown in figure two. By hedging, the manager may increase the weight in the left tail of the corporate payoff and decrease the weight in the right tail as shown in figure two. Letting η be the state in which there is no capital gain or loss on the forward position, β be boundary of the in the money event for the unhedged firm and α be the boundary of the in the money event for the hedged firm, the figure depicts the clearest case of a loss in option value due to the hedge.

Figure 2: Hedging with Forwards



Hedging Theorem.[15] Let the payoff on the stock index Φ be increasing in state. *Ceteris paribus*, the manager paid in stock options does not have an incentive to hedge in forward contracts written on the stock index.

This theorem provides an alternative explanation for why managers paid in stock options do not hedge. Hedging in forward contracts reduces the weight in the right tail of the corporate payoff distribution and this has the effect of reducing the value of the warrants. The result is similar to that in Smith and Stulz (Smith and Stulz 1985) but the model is not. In Smith and Stulz, the manager considers hedging due to risk aversion and there is no separation between decision on personal and corporate account; there the result followed when the convexity introduced by the stock option package was enough to counteract the concavity of the utility function, i.e., the risk aversion. (Tufano 1996) notes ". . . the data bear out Smith and Stutz's (1985) prediction that firms whose managers own more stock options manage less gold price risk, and those whose managers have more wealth invested in common stock manage more gold price risk."[16] There are other theories to explain this. With the Fisher separation here the result follows more generally because a hedge in forward contracts reduces the value of the warrants. It does not have to be risk aversion that is driving it.

[15] See the appendix for additional discuss and the proof of this theorem.

[16] See p. 1099 in Tufano, P. (1996). "Who Manges Risk? An Emprirical Examination of Risk Management Practices in the Gold Mining Industry." Journal of Finance **51**(4): 1097-1137.

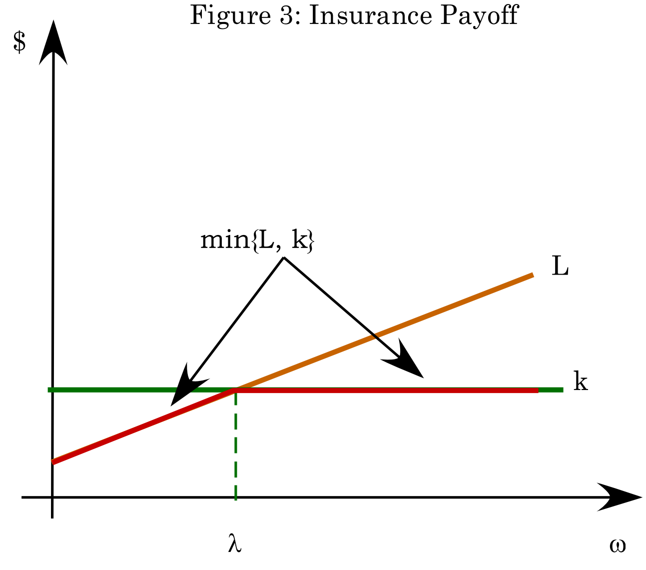
A manager paid in stock options wants more weight in the right tail because the manager maximizes the warrant value in the compensation package. Indeed it is not apparent that Tufano's empirical results can distinguish between the two models.

The Insured Corporation

In the last section, the manager chose to over-invest and given the ability to manage risk with forward contracts chose not to manage the risk. An insurance contract is introduced here and first we consider whether the manager has an incentive to purchase insurance on corporate account and if so why. Second, we consider how insurance changes the investment incentives.

To explicate the points made here suppose that the losses are liability losses and that the contract provides indemnity up to a cap of k dollars. Then the insurance contract payoff is min{L, k}. Recall that the corporate payoff net of insurable losses, i.e., Π - L, is assumed to be an increasing function of state. This can be a result of many combinations of the payoff function and loss function. Two cases might be considered. The first case is provided in figure three. It is supposed there that the loss increases in state and incorporates the notion that losses increase with economic activity. The second case of a decreasing loss function is provided in the appendix. Let λ denote the state where the loss is equal to the cap. Given the increasing loss, the insured corporation gets k dollars from the insurer as indemnity and absorbs the loss over and beyond k in states greater than λ. Letting i(k) denote the market value of the insurance contract we may note that

Figure 3: Insurance Payoff



$$
\begin{aligned}
i(k) &= \int_0^{\zeta} \min\left\{L\left(I,\omega\right),k\right\} dP \\
&= \int_0^{\lambda} L\left(I,\omega\right) dP + \int_{\lambda}^{\zeta} k\, dP
\end{aligned}
\tag{9}
$$

Observe that the corporate value of the uninsured corporation is V^u where

$$V^u = \int_0^{\zeta} (\Pi - L) dP \qquad (10)$$

while the value of the insured corporation is V^i where

$$\begin{aligned} V^i &= -i + \int_o^{\zeta} \left(\Pi - L + \min\{L, k\}\right) dP \\ &= V^u \end{aligned} \qquad (11)$$

The second equality in (11) follows by (9) and provides a familiar corollary to the 1958 Modigliani-Miller theorem (Modigliani and Miller 1958); for examples in the literature see (MacMinn 1987). Equation (11) provides a clear corollary to the theorem and shows that insurance *ceteris paribus* does not add value. Equivalently, it may be shown that the stock market value increases equal to that of the increase in the insurance premium so that again no value is added. A gain or loss in value will follow, however, if the investment level is affected by the insurance decision.

Now consider the value of the manager's warrant given the possible insurance coverage. The warrant payoff becomes

$$\begin{aligned} &\max\left\{0,\ a\left(\Pi - L + \min\{L, k\} + e\,m - b\right) - e\,m\right\} = \\ &\max\left\{0,\ a\left(\Pi - L + \min\{L, k\} - b\right) - (1 - a)E\right\} \end{aligned} \qquad (12)$$

The warrant value becomes W(I, k) where

$$W(I, k) = \int_0^{\zeta} \max\left\{0,\ a\left(\Pi - L + \min\{L, k\} - b\right) - (1 - a)E\right\} dP \qquad (13)$$

The financing condition in the constrained maximization problem must be altered to raise enough now to cover the insurance premium as well as the investment level; the financing constraint becomes

$$D(b) \equiv \int_0^{\zeta} b\ dP = I + i(k) \qquad (14)$$

but it continues to implicitly define the function b(I, k). As previously, suppose that any debt issue is safe. Then let F(I, k, b) = 0 implicitly define b(I, k) where

$$\begin{aligned} F(I, k, b) &= D(b) - I - i(k) \\ &= \int_0^{\zeta} b\, dP - I - \left(\int_0^{\lambda} L(I, \omega)\, dP + \int_{\lambda}^{\zeta} k\, dP\right) \end{aligned} \qquad (15)$$

The derivative properties of the function b(I, k) are provided in the following lemma.

Lemma 1. The promised payment on the debt issue increases in the investment I and the cap k.

Proof. By the Implicit Function Theorem

$$\frac{\partial b}{\partial I} = -\frac{\frac{\partial F}{\partial I}}{\frac{\partial F}{\partial b}} = \frac{1 + \int_0^{\lambda} \frac{\partial L}{\partial I} dP}{\int_0^{\zeta} dP} > 0 \tag{16}$$

and

$$\frac{\partial b}{\partial k} = -\frac{\frac{\partial F}{\partial k}}{\frac{\partial F}{\partial b}} = \frac{\int_{\lambda}^{\zeta} dP}{\int_0^{\zeta} dP} > 0 \tag{17}$$

QED

Now consider the manager's insurance decision in isolation. Maintain the assumption that the manager is compensated with stock options. Then the manager will choose to insure if W(I, k) > W(I,0) for k > 0.

To motivate the comparison of warrant values consider figure four.[17] The line labeled with $a\left(\Pi(I,\omega) - L(I,\omega) - b(I,0)\right)$ represents the gross warrant payoff in the uninsured case. The line labeled with $a\left(\Pi(I,\omega) - L(I,\omega) - b(I,k)\right)$ is the payoffs prior to the indemnity from the insurer. Note that b(I, k) is the promised debt payment required to pay for the investment and insurance. The line labeled $a\left(\Pi(I,\omega) - L(I,\omega) + \min\left\{L(I,\omega),k\right\} - b(I,k)\right)$ represents the gross warrant payoff after receipt of insurance indemnity. The gross insured warrant payoff falls below the gross uninsured warrant payoff for some states but above for others. The gross insured warrant payoff is greater than the uninsured counterpart for the larger states and the manager benefits because those are the in the money states for the options; the gross uninsured payoff is greater than the insured counterpart for smaller states but those correspond more to the out of the money event and so do not adversely affect the manager. To motivate this comparison this comparison of gross warrant payoffs, note that the insurance decision yields more weight in the right tail of the payoff distribution if

$$\Pi(I,\omega) - L(I,\omega) + \min\left\{L(I,\omega),k\right\} - b(I,k) > \Pi(I,\omega) - L(I,\omega) - b(I,0) \tag{18}$$

For states $\omega > \lambda$, (18) becomes

$$\Pi(I,\omega) - L(I,\omega) + k - b(I,k) > \Pi(I,\omega) - L(I,\omega) - b(I,0) \tag{19}$$

[17] The relative positions of α, β, and λ in figure three are just for illustration and do not to affect the result in theorem two.

or equivalently

$$k > b(I,k) - b(I,0) \tag{20}$$

Given the safe debt issue, the financing condition in (14) yields $b(I,k) = \left(I + i(k)\right) \Big/ \int_0^{\zeta} dP$ and so (20) may be equivalently expressed as

$$k > \frac{i(k)}{\int_0^{\zeta} dP}$$

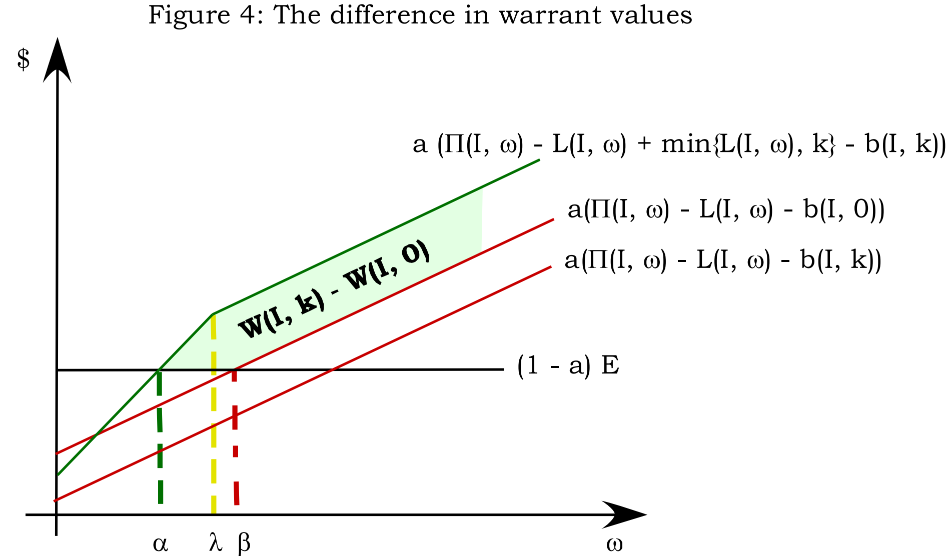
$$\Leftrightarrow \int_0^{\zeta} k\, dP > \int_0^{\lambda} L\left(I,\omega\right) dP + \int_{\lambda}^{\zeta} k\, dP \tag{21}$$

$$\Leftrightarrow \int_0^{\lambda} k\, dP > \int_0^{\lambda} L\left(I,\omega\right) dP$$

The last inequality in (21) clearly holds and so establishes (19) for all $\omega > \lambda$ and for some $\omega < \lambda$ by continuity. Hence, the insurance increases the payoff in the right tail of the corporate payoff distribution and reduces it in the left tail as shown in figure four.

The warrant payoffs are shown in the next figure. The shaded area in the figure represents W(I, k) - W(I,0) and shows that the manager has a motivation to insure. The positive intercepts simply indicate that the debt is safe. As long as the manager can optimally select an investment and finance that investment with safe debt, the motivation to insure exists. The incentive to insure also follows from the first order condition and is expressed in the following theorem.

Figure 4: The difference in warrant values



Theorem 2. Let Π, L and Π - L be increasing in state.[18] *Ceteris paribus*, the manager paid in stock options has an incentive to insure the liability losses.

[18] This theorem demonstrates what we consider the more common case that as the corporate payoff increases so does the liability loss. The case in which the corporate payoff Π increases and the liability losses decrease is in the appendix.

Proof. Consider two cases.

Case I: $0 < \alpha < \lambda$

Here the warrant value is

$$W(I,k) = \int_{\alpha}^{\lambda} \left[a\left(\Pi - b\right) - \left(1 - a\right)E \right] dP + \int_{\lambda}^{\zeta} \left[a\left(\Pi - \left(L - k\right) - b\right) - \left(1 - a\right)E \right] dP \qquad (22)$$

and the first order condition for k is

$$\begin{aligned}
\frac{\partial W}{\partial k} &= a\left(\int_{\alpha}^{\lambda} \left(-\frac{\partial b}{\partial k} \right) dP + \int_{\lambda}^{\zeta} \left(1 - \frac{\partial b}{\partial k} \right) dP \right) \\
&= a\left\{ \int_{\lambda}^{\zeta} dP - \frac{\partial b}{\partial k} \int_{\alpha}^{\zeta} dP \right\} \\
&= a\left\{ \int_{\lambda}^{\zeta} dP - \frac{\int_{\lambda}^{\zeta} dP}{\int_{0}^{\zeta} dP} \int_{\alpha}^{\zeta} dP \right\} \qquad (23) \\
&= a \int_{\lambda}^{\zeta} dP \left\{ 1 - \frac{\int_{\alpha}^{\zeta} dP}{\int_{0}^{\zeta} dP} \right\} \\
&> 0
\end{aligned}$$

Case II: $\alpha > \lambda > 0$

Here the warrant value is

$$W(I,k) = \int_{\alpha}^{\zeta} \left[a\left(\Pi - \left(L - k\right) - b\right) - \left(1 - a\right)E \right] dP \qquad (24)$$

and the first order condition for k is

$$\frac{\partial W}{\partial k} = a\int_{\alpha}^{\zeta}\left(1-\frac{\partial b}{\partial k}\right)dP$$

$$= a\left\{\int_{\alpha}^{\zeta}dP - \frac{\int_{\lambda}^{\zeta}dP}{\int_{0}^{\zeta}dP}\int_{\alpha}^{\zeta}dP\right\} \tag{25}$$

$$= a\int_{\alpha}^{\zeta}dP\left\{1-\frac{\int_{\lambda}^{\zeta}dP}{\int_{0}^{\zeta}dP}\right\}$$

$$> 0$$

QED

Since the insurance does not change the corporate value and the debt issue is appropriately priced it follows that there is a transfer of value from shareholders to the manager. Note that the manager paid with stock options has the incentive insure if the optimal investment level can be selected and financed with safe debt. The incentive to insure exists because the insurance places more weight in the right tail of the corporate earnings distribution. Not all hedging instruments are created equally. The forward contracts do reduce risk but do it by moving weight from the tails of the earnings distribution; since weight is removed from the right tail the manager has no incentive to hedge with forward contracts but does have the incentive to hedge with insurance contracts.

Next, consider the second objective, i.e., what impact does the insurance have on the manager's investment choice? Does the investment choice become a function of the cap on the insurance? To answer the first question, consider the impact that insurance has on the manager's investment decision by comparing the optimal investment in the uninsured and insured cases. The manager with insurance coverage selects the investment level to maximize the warrant value expressed in (13). Let I^{W^i} and I^{W^u} denote the manager's optimal investment choices in the insured and uninsured cases respectively. The next theorem demonstrates the relation between these investment choices.

Theorem 4. Let Π, L and Π - L be increasing in state and let the PIU hold. Insurance reduces the manager's incentive to over-invest, i.e., $I^{W^u} > I^{W^i}$

Consider the following two cases.

Case I: Let α and β denote the boundaries of the option exercise events in the insured and uninsured cases respectively, as shown in figure four. As in the figure, let $\alpha < \lambda < \beta$. The first order condition in the insured case may be expressed as follows:

$$\frac{\partial W(I,k)}{\partial I} = \int_{\alpha}^{\lambda} a\left(\frac{\partial \Pi}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right)dP + \int_{\lambda}^{\zeta} a\left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right)dP = 0 \tag{26}$$

Similarly, the first order condition in the uninsured case may be expressed as follows:

$$\frac{\partial W(I,0)}{\partial I} = \int_{\beta}^{\zeta} a\left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,0)}{\partial I}\right) dP = 0 \tag{27}$$

The difference in the derivatives is

$$\begin{aligned}
\left.\frac{\partial W(I,k)}{\partial I} - \frac{\partial W(I,0)}{\partial I}\right|_{I=I^{W^i}} &= \int_{\alpha}^{\lambda} a\left(\frac{\partial \Pi}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right) dP + \int_{\lambda}^{\zeta} a\left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right) dP \\
&\quad - \int_{\beta}^{\zeta} a\left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,0)}{\partial I}\right) dP \\
&= \int_{\alpha}^{\lambda} a\left(\frac{\partial \Pi}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right) dP + \int_{\lambda}^{\beta} a\left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right) dP \\
&\quad + \int_{\beta}^{\zeta} a\left(\frac{\partial b(I,0)}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right) dP \\
&= \int_{\alpha}^{\lambda} a\left(\frac{\partial \Pi}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right) dP + \int_{\lambda}^{\beta} a\left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right) dP \\
&\quad - a\frac{\int_0^{\lambda} \frac{\partial L}{\partial I} dP}{\int_0^{\zeta} dP} \int_{\beta}^{\zeta} dP \\
&< 0
\end{aligned} \tag{28}$$

The third equality follows by observing that

$$\frac{\partial b(I,0)}{\partial I} = \frac{1}{\int_0^{\zeta} dP}$$

since λ is zero. Recall that the PIU shows that $\left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b}{\partial I}\right)$ is increasing. Since it is also negative for some ω and monotone increasing, when we eliminate the integrand from β to ζ we have eliminated part of the positive component and left the negative. Hence the difference in derivatives in (28) is negative by the PIU. Therefore, the uninsured investment level exceeds the insured level, i.e., $I^{W^u} > I^{W^i}$.

Case II: Let λ < α < β. The first order condition for the investment level in the insured case is

$$\frac{\partial W(I,k)}{\partial I} = \int_{\alpha}^{\zeta} a\left(\frac{\partial \Pi}{\partial I} - \frac{\partial L}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right) dP = 0 \tag{29}$$

and from (29) and (27) the difference in first order conditions is

$$\left.\frac{\partial W(I,k)}{\partial I}-\frac{\partial W(I,0)}{\partial I}\right|_{I=I^{W^i}} = \int_\alpha^\zeta a\left(\frac{\partial \Pi}{\partial I}-\frac{\partial L}{\partial I}-\frac{\partial b(I,k)}{\partial I}\right)dP - \int_\beta^\zeta a\left(\frac{\partial \Pi}{\partial I}-\frac{\partial L}{\partial I}-\frac{\partial b(I,0)}{\partial I}\right)dP$$

$$= \int_\alpha^\beta a\left(\frac{\partial \Pi}{\partial I}-\frac{\partial L}{\partial I}-\frac{\partial b}{\partial I}\right)dP + \int_\beta^\zeta a\left(\frac{\partial b(I,0)}{\partial I}-\frac{\partial b(I,k)}{\partial I}\right) \tag{30}$$

$$= \int_\alpha^\beta a\left(\frac{\partial \Pi}{\partial I}-\frac{\partial L}{\partial I}-\frac{\partial b}{\partial I}\right)dP - a\frac{\int_0^\lambda \frac{\partial L}{\partial I}dP}{\int_0^\zeta dP}\int_\beta^\zeta dP$$

$$< 0$$

The inequality in (30) follows by the PIU and so, as in case I, the uninsured investment level exceeds the insured level, i.e., $I^{W^u} > I^{W^i}$ QED

Theorem two shows that there is an incentive to insure. Theorem four shows that the insurance reduces the incentive to over-invest.

Some questions remain. Can insurance be used to effectively eliminate the over-investment problem? Does the optimal investment get smaller as the insurance coverage is increased? The next theorem shows how increasing insurance coverage affects the manager's investment decision.

Theorem 5. Let Π, L and Π - L be increasing in state. The optimal investment increases with added insurance coverage if $\alpha < \lambda$ and decreases with added insurance coverage if $\alpha > \lambda$.

Proof. Let I(k) denote the manager's optimal investment decision given the insurance cap k. Since the optimal investment is implicitly defined by

$$\frac{\partial W(I,k)}{\partial I} = 0 \tag{31}$$

It follows by implicit differentiation that

$$\frac{dI}{dk} = -\frac{\dfrac{\partial^2 W}{\partial k\,\partial I}}{\dfrac{\partial^2 W}{\partial^2 I}} \tag{32}$$

Hence, the optimal investment increases or decreases in k as the cross partial in the numerator of (32) is positive or negative, respectively.

Case I: If $\alpha > \lambda$ then the warrant value is given in (24). The marginal warrant value for this case is expressed in equation (29) and differentiating again with respect to coverage yields

$$\frac{\partial^2 W(I,k)}{\partial k\,\partial I} = -a\left(\frac{\partial \Pi(I,\alpha)}{\partial I} - \frac{\partial L(I,\alpha)}{\partial I} - \frac{\partial b(I,k)}{\partial I}\right) p(\alpha)\frac{\partial \alpha}{\partial k} < 0 \tag{33}$$

The inequality follows since α is a decreasing function of the cap k and the marginal equity payoff in parentheses is negative at α.[19] Hence, the optimal investment decreases as the cap increases.

Case II: If $\alpha < \lambda$ then the warrant value is expressed in (22) and the marginal warrant value is expressed in (26). Differentiating (26) with respect to k yields

$$\begin{aligned}\frac{\partial^2 W(I,k)}{\partial k\,\partial I} &= a\left(\frac{\partial \Pi(I,\lambda)}{\partial I} - \frac{\partial b}{\partial I}\right) p(\lambda)\frac{\partial \lambda}{\partial k} \\ &\quad - a\left(\frac{\partial \Pi(I,\lambda)}{\partial I} - \frac{\partial L(I,\lambda)}{\partial I} - \frac{\partial b}{\partial I}\right) p(\lambda)\frac{\partial \lambda}{\partial k} \\ &= a\frac{\partial L(I,\lambda)}{\partial I} p(\lambda)\frac{\partial \lambda}{\partial k} \\ &> 0\end{aligned} \tag{34}$$

Hence, in this case, the optimal investment increases as the cap increases. QED

Note that case I results when the exercise price is sufficiently large. Without violating the case I condition, an increase in the cap reduces the set in which the warrant is out of the money, thus increasing the warrant value. As a result, the manager has less incentive to resort to over-investment as a tool to raise the warrant value. The case II condition results when the exercise price is sufficiently small so that the “at the money” state α is smaller than the full insurance state λ. This is the case depicted in figure four; an increase in k does not change the out-of-the-money set. That is, raising k *per se* does not increase the warrant value. Over-investment is left as the manager’s only

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[19] The negative sign of the marginal equity payoff follows due to the PIU. Note that the marginal equity payoff is monotone increasing by the PIU and so takes negative values followed by positive values.

tool to increase the warrant value. Even so, theorem four shows that insurance still reduces the over-investment motivated by stock option compensation schemes.

Theorem five clearly shows that the CEO's investment choice is influenced by the insurance coverage. The second case in theorem five shows that the insurance may either increase or decrease the investment incentive but may well be moot since theorem two shows that the warrant value can be increased with insurance coverage. In combination, theorems two and five show that insurance can be an effective tool in reducing the adverse incentives created by the stock option compensation scheme.

Concluding Remarks

With few exceptions the literature on the demand for corporate insurance either implicitly or explicitly assumes that firm decisions are made to maximize the stock value. The connection between management compensation and corporate decision making has not been adequately explored.[20] Stock options have become an increasingly important component of compensation packages in part due to the conventional but flawed wisdom that they align management and shareholder incentives. This analysis has shown that the flaw becomes evident in the objective function provided by the separation theorem.

Smith and Stulz (1985) show that managers paid in stock options will ultimately not hedge corporate risk. The manager in their analysis maximizes the expected utility of the end of period wealth due to the compensation with no ability to diversify on personal account; this eliminates the possibility of a separation result. This model allows the manager to make decisions on corporate account and personal account. The model allows the endogenous derivation of the objective function used by the manager in making decisions on corporate account. The objective function shows that the manager will over-invest relative to the investment that maximizes current shareholder value or equivalently net present value and that the manager will not manage the corporate risk with forward contracts. Risk aversion drove the hedging decision in the Smith and Stulz analysis. In this setting the manager paid in stock would be on the razor's edge of indifference and the manager paid in stock options would see a reduction in the option value with any hedge in forwards and so would eschew that risk management tool.

Hedging with forwards creates a symmetrical reduction in volatility. Insurance, however, does not reduce volatility in the same way. Insurance involves a risk transfer from the insured to the insurer for a price. The loss and thus indemnity can be higher, lower, or equal to the future value of the insurance premium. Insurance can transfer cash from states with small losses to states with large losses. If the transfer moves cash from the out-of-the-money states to the in-the-money states then the manager benefits and thus has an incentive to insure. Hence, not all risk management tools are created equally. The analysis here shows a case in which the manager will insure despite having no incentive to hedge with forward contracts.

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[20] There is an empirical literature that attempts to make this connection but the results are ambiguous or weak; the ambiguity may well arise because the incentives arising from a bonus scheme may be different from those arising in a restricted stock or a stock option scheme. Those differences are not typically considered in the empirical literature and have not been adequately explored in the theoretical literature. On bonuses see Brander, J. A. and M. Poitevin (1992). "Managerial Compensation and the Agency Costs of Debt Finance." Managerial and Decision Economics: 55-64. and MacMinn, R. (1992). Lecture on Managerial Compensation and Agency Costs.

Finally, this analysis shows that managers paid in stock options over-invest to maximize their stock option values at the expense of shareholders and that insurance is irrelevant to the value of the corporation, holding investment constant. The manager's gain from insuring is the shareholders' loss *ceteris paribus*. However, when managers insure on corporate account, the over-investment problem induced by stock option compensation is reduced. Hence, while the shareholder will lose from the manager's insuring on corporate account, that loss will be offset by the closer-to-optimal investment that the manager is motivated to make.

This study treats stock options' exercise price as given. Bebchuk, Fried *et al.* (2002) argue that setting the exercise price reflects the agency problem. Theorem five supports this view to the extent that it shows that a sufficiently large exercise price in conjunction with insurance reduces the over-investment. Research taking into consideration the interaction among exercise price, corporate investment and insurance decisions can further add to our understanding of the effects of stock option compensation on corporate decisions.

Appendix

Fisher Separation Theorem. The manager paid in stock options *now* that vest *then* makes decisions on corporate account that are independent of the manager's preferences for consumption now versus then; the decisions are made to maximize the warrant value W(I).

Prof. Let n denote the number of states of nature and $c_1 = \left(c_1(\omega_1), c_1(\omega_2), \ldots, c_1(\omega_n)\right)$ denote the consumption *then* vector. Let $L\left(c_0, c_1, I, \theta\right)$ be the Lagrange function corresponding to the constrained maximization problem in (3) where θ is the Lagrange multiplier.

$$\begin{aligned} L(c_0,\, c_1, I, \theta) = & \int_\Omega u\left(c_0,\, c_1(\omega)\right)\, d\Psi(\omega) \\ & -\theta\Big(c_0 + \int_\Omega c_1(\omega)\, dP(\omega) - m_0 - \int_\Omega m_1(\omega)\, dP(\omega) - W(I)\Big) \end{aligned} \qquad (35)$$

The first order conditions for a maximum are:

$$\frac{\partial L}{\partial c_0} = \int_\Omega \frac{\partial u}{\partial c_0}\, d\Psi(\omega) - \theta = 0 \qquad (36)$$

$$\frac{\partial L}{\partial c_1(\omega_j)} = \frac{\partial u}{\partial c_1(\omega_j)} \psi(\omega_j) - \theta\, p(\omega_j) = 0, j = 1, \ldots, n \tag{37}$$

$$\frac{\partial L}{\partial I} = -\theta \frac{dW}{dI} = 0 \tag{38}$$

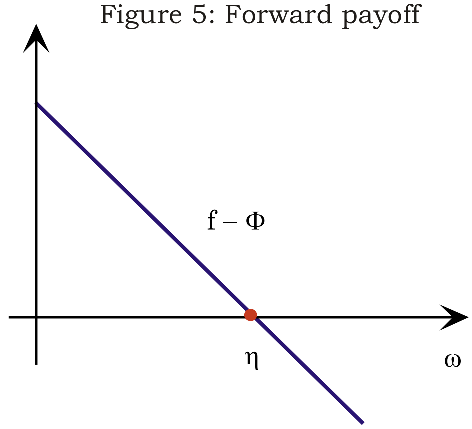
and

$$\frac{\partial L}{\partial \theta} = -\left(c_0 + \int_\Omega c_1(\omega)\, dP(\omega) - m_0 - \int_\Omega m_1(\omega)\, dP(\omega) - W(I)\right) = 0 \tag{39}$$

Equations (36) and (37) express the conditions for the optimal consumption pair while (38) expresses the condition for an optimal investment. Hence, the Fisher separation result follows since (38) is independent of preferences for consumption *now* and *then*; in this uncertainty setting it should also be noted that (38) is independent of the manager's measure of risk aversion and probability beliefs. Clearly the objective function that the manager uses to make the investment decision on corporate account is the value of the stock option package. QED

Next consider the corporate hedging decision. The payoff on the forward market position taken by the corporation is shown in the figure. Let Φ denote the unit payoff on a stock index forward contract and let f denote the forward price. Suppose the unit payoff is increasing in state. The payoff on the forward position is φ (f - $\Phi(\xi)$), where φ is the position taken by the firm in forwards. Let α be the economic state implicitly defined by the condition f - $\Phi(\eta)$ = 0 as shown in figure five. The payoff depicted is sometimes referred to as a short position in the futures contract. If the corporation selects φ units of the hedge then the corporate payoff is $\Pi - L + \varphi(f - \Phi) - b$ as shown in figure two. The following hedging theorem follows easily.

Figure 5: Forward payoff



Hedging Theorem. Let the payoff on the stock index Φ be increasing in state. *Ceteris paribus*, the manager paid in stock options does not have an incentive to hedge in forward contracts written on the stock index.

Proof. The manager's objective function for decision made on corporate account is the warrant value $W(I,\varphi)$ where

$$\begin{aligned} W(I,\varphi) &= \int_0^{\zeta} \max\left\{0, a\left(\Pi - L + \varphi\left(f - \Phi\right) - b\right) - \left(1 - a\right)E\right\} dP \\ &= \int_{\alpha}^{\zeta} \left(a\left(\Pi - L + \varphi\left(f - \Phi\right) - b\right) - \left(1 - a\right)E\right) dP \end{aligned} \tag{40}$$

The first order condition for the forward contracts is

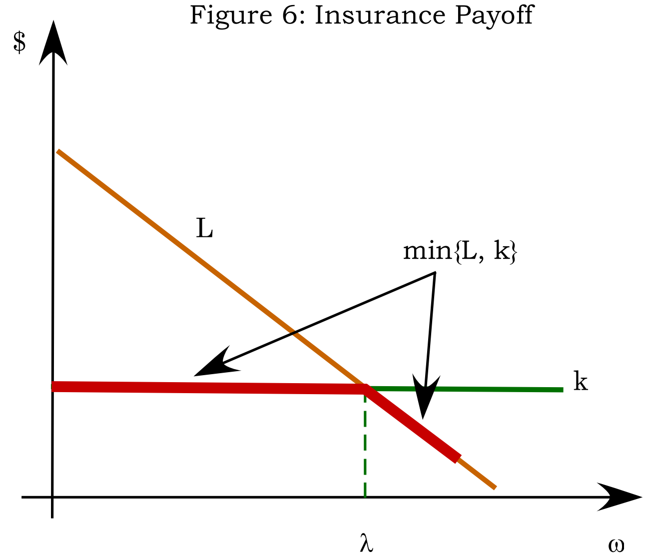
$$\frac{\partial W}{\partial \varphi} = a\int_{\alpha}^{\zeta}\left(f - \Phi\right) dP < 0 \tag{41}$$

The inequality follows since letting $H(\alpha) = \int_{\alpha}^{\zeta}\left(f - \Phi\right) dP$ it may be observed that that H(0) = 0 and H is decreasing. QED

Next consider the case of a liability loss that is decreasing in state. The specification of the insurance premium changes as may be seen in figure six. The insurance payoff is now

$$\begin{aligned} i(k) &= \int_0^{\zeta} \min\left\{L\left(I,\omega\right), k\right\} dP \\ &= \int_0^{\lambda} k \, dP + \int_{\lambda}^{\zeta} L\left(I,\omega\right) dP \end{aligned} \tag{42}$$

Figure 6: Insurance Payoff



Hence, the promised payment on debt as a function of the investment must be implicitly specified as

$$\begin{aligned} G\left(I,k,b\right) &= D(b) - I - i(k) \\ &= \int_0^{\zeta} b\,dP - I - \left(\int_0^{\lambda} k\,dP + \int_{\lambda}^{\zeta} L\left(I,\omega\right)dP\right) \end{aligned} \tag{43}$$

Lemma 2. Given the loss that decreases in state, the promised payment on the debt issue increases in the investment I and the cap k.

Proof. By the Implicit Function Theorem

$$\frac{\partial b}{\partial I} = -\frac{\frac{\partial G}{\partial I}}{\frac{\partial G}{\partial b}} = \frac{1 + \int_{\lambda}^{\zeta} \frac{\partial L}{\partial I} dP}{\int_0^{\zeta} dP} > 0 \tag{44}$$

and

$$\frac{\partial b}{\partial k} = -\frac{\frac{\partial G}{\partial k}}{\frac{\partial G}{\partial b}} = \frac{\int_0^{\lambda} dP}{\int_0^{\zeta} dP} > 0 \tag{45}$$

QED

Theorem 3. Let Π and Π - L be increasing in state; let L be decreasing in state. *Ceteris paribus*, the manager paid in stock options has an incentive not to insure the liability losses if α > λ; otherwise the incentive is ambiguous.

Proof. Consider the case in which the losses are decreasing in state as shown in figure five.

$$\Pi - L + \min\{L,k\} = \begin{cases} \Pi - (L-k) & \omega \le \lambda \\ \Pi & \omega > \lambda \end{cases} \tag{46}$$

Then the following two case must be considered.

Case I: $\alpha < \lambda$

The warrant value in this case is

$$\begin{aligned} W(I,k) = &\int_{\alpha}^{\lambda}\Big[a\big(\Pi - (L-k) - b\big) - (1-a)E\Big]\, dP \\ &+ \int_{\lambda}^{\zeta}\Big[a(\Pi - b) - (1-a)E\Big]dP \end{aligned} \tag{47}$$

and

$$\begin{aligned} \frac{\partial W}{\partial k} &= a\int_{\alpha}^{\lambda}\left(1 - \frac{\partial b}{\partial k}\right) dP - a\int_{\lambda}^{\zeta}\frac{\partial b}{\partial k} dP \\ &= a\left(\int_{\alpha}^{\lambda} dP - \frac{\partial b}{\partial k}\int_{\alpha}^{\zeta} dP\right) \\ &= a\left(\int_{\alpha}^{\lambda} dP - \frac{\int_{0}^{\lambda} dP}{\int_{0}^{\zeta} dP}\left(\int_{\alpha}^{\lambda} dP + \int_{\lambda}^{\zeta} dP\right)\right) \\ &= a\left(\int_{\alpha}^{\lambda} dP\left(1 - \frac{\int_{0}^{\lambda} dP}{\int_{0}^{\zeta} dP}\right) - \frac{\int_{0}^{\lambda} dP}{\int_{0}^{\zeta} dP}\int_{\lambda}^{\zeta} dP\right) \end{aligned} \tag{48}$$

The first term on the right hand size of the last equality is positive but the sign of the whole term is indeterminate.

Case II: $\alpha > \lambda$

Here the warrant value is

$$W(I,k) = \int_{\alpha}^{\zeta} \left[a\left(\Pi - b\right) - \left(1 - a\right)E \right] dP \qquad (49)$$

and

$$\frac{\partial W}{\partial k} = -a\frac{\partial b}{\partial k}\int_{\alpha}^{\zeta} dP < 0 \qquad (50)$$

QED

References

Acharya, S. (1992). "Maximizing the Market Value of a Firm to Choose Dynamic Policies for." International Economic Review: 373-97.

Acharya, V. V., K. John, et al. (2000). "On the Optimality of Resetting Executive Stock Options." Journal of Financial Economics: 65-101.

Bebchuk, J., M. Fried, et al. (2002). "Managerial Power and Rent Extraction in the Design of Executive Compensation." The University of Chicago Law Review: 751 - 856.

Brander, J. A. and M. Poitevin (1992). "Managerial Compensation and the Agency Costs of Debt Finance." Managerial and Decision Economics: 55-64.

Carpenter, J. N. (2000). "Does Option Compensation Increase Managerial Risk Appetite?" Journal of Finance **55**(5): 2311-2331.

Chidambaran, N. K. and N. R. Prabhala (2003). "Executive Stock Option Repricing, Internal Governance Mechanisms, and Management Turnover." Journal of Financial Economics: 153-189.

DeFusco, R. A., R. R. Johnson, et al. (1990). "The Effect of Executive Stock Option Plans on Stockholders and Bondholders." Journal of Finance **45**(2): 617-27.

Garven, J. R. and R. D. MacMinn (1993). "The Underinvestment Problem, Bond Covenants and Insurance." Journal of Risk and Insurance.

Green, R. C. (1984). "Investment Incentives, Debt and Warrants." Journal of Financial Economics **13**: 115-36.

Han, L.-M. (1996). "Managerial Compensation and Corporate Demand for Insurance." Journal of Risk and Insurance: 381-404.

Leland, H. (1972). "Theory of the Firm Facing Uncertain Demand." American Economic Review **62**: 278-91.

Lewellen, W. e. a. (1992). "Executive Compensation and the Performance of the Firm." Managerial and Decision Economics: 65-74.

MacMinn, R. (1992). Lecture on Managerial Compensation and Agency Costs.

MacMinn, R. and F. Page (1995). "Stock Options, Managerial Incentives, and Capital Structure." Journal of Financial Studies.

MacMinn, R. D. (1987). "Insurance and Corporate Risk Management." Journal of Risk and Insurance **54**(4): 658-77.

MacMinn, R. D. (1993). On The Risk Shifting Problem and Convertible Bonds. Advances in Quantitative Analysis of Finance and Accounting.

MacMinn, R. D. and L. M. Han (1990). "Limited Liability, Corporate Value, and the Demand for Liability Insurance." Journal of Risk and Insurance **57 4**: 581-607.

MacMinn, R. D. and A. Holtmann (1983). "Technological Uncertainty and the Theory of the Firm." Southern Economic Journal **50**: 120-36.

Main, B. G. (1983). "Corporate Insurance Purchases and Taxes." Journal of Risk and Insurance: 197-223.

Main, B. G. M. (1991). "Top Executive Pay and Performance." Managerial and Decision Economics **12**(3): 219-29.

Mayers, D. and C. Smith (1982). "On the Corporate Demand for Insurance." Journal of Business **55**: 281-96.

Mayers, D. and C. W. Smith, Jr. (1992). "Executive Compensation in the Life Insurance Industry." Journal of Business: 51-74.

Mayers, D. and C. W. J. Smith (1987). "Corporate Insurance and the Underinvestment Problem." Journal of Risk and Insurance: 45-54.

Merton, R. C. (1973). "Theory of Rational Option Pricing." Bell Journal of Economics and Management Science **4**(1): 141-83.

Modigliani, F. and M. H. Miller (1958). "The Cost of Capital, Corporation Finance and the Theory of Investment." American Economic Review **48**(2): 261-97.

Murphy, K. J. (1998). Executive Compensation. University of Southern California, Los Angeles.

Murphy, K. J. (1999). Executive Compensation. Handbook of labor economics. D. Card. Handbooks in Economics, vol. 5. Amsterdam; New York and Oxford, Elsevier Science North-Holland. **3B:** 2485-2563.

Smith, C. W. and R. M. Stulz (1985). "The Determinants of Firms' Hedging Policies." Journal of Financial and Quantitative Analysis **20**(4): 391-405.

Tufano, P. (1996). "Who Manges Risk? An Emprirical Examination of Risk Management Practices in the Gold Mining Industry." Journal of Finance **51**(4): 1097-1137.