On Corporate Risk Management and Insurance

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August 1994
Revised September 1996
Revised August 1997
Revised December 1998
Revised June 1999

Comments, on an earlier draft of this work, from the participants at the 1996 Hannover, Germany, Meeting of the Geneva Association of Risk and Insurance Economists are gratefully acknowledged. Thanks for comments also goes to the participants at the 1997 American Risk and Insurance Association Meetings in San Diego, the 1997 Inaugural Meeting of the Asia-Pacific Risk and Insurance Association Meetings in Singapore and the 1998 Western Risk and Insurance Association Meetings. A summer research grant from the Gus S. Wortham Chair at the University of Texas partially funded this research and is gratefully acknowledged here.

Introduction

One of the primary purposes of a financial market system is to facilitate the allocation of risk bearing. A competitive financial market system can generate an efficient allocation of risk bearing and so an efficient allocation of resources. Insurance contracts compose a subset of the contracts traded in a financial market system. Like other financial contracts, the purpose of an insurance contract is to affect a risk transfer that results in a redistribution of the risk. Like other financial contracts, the insurance contract does generate an allocation of risk bearing among the market agents but unlike other contracts the insurance contract often affects a complete transfer of risk rather than a partial transfer. Financial market theory contains no paradigm that explains the existence of the insurance mechanism.[1] Indeed, a generalization of the 1958 Modigliani-Miller theorem shows that corporate insurance is irrelevant.[2] That theorem, however, also shows that financial leverage is irrelevant and implies that hedging with futures or forward contracts is irrelevant. Therefore, there is a need to modify the current finance paradigm[3] so that the existence and function of the insurance mechanism can be analyzed. The current paradigm is modified here and the question of an optimal mix of financial contracts is addressed in a market that includes bond, stock, insurance, and futures contracts. The insurance contract plays an integral role in the determination of the corporation's optimal nexus of contracts; in fact, the analysis shows that the corporation has an incentive to fully insure property risks and to do so by levering. Hence, an optimal capital structure requires insurance. The analysis also shows that the firm does not have an incentive to use futures contracts in isolation since a full hedge does not capture all the value that can be captured with insurance or, more generally, a combination of futures and insurance.

A number of authors have considered corporate insurance (Cummins 1976; Mayers and Smith 1982; Main 1983; Shapiro and Titman 1985; MacMinn 1987; Mayers and Smith 1987; MacMinn 1989; Garven and MacMinn 1993) This literature provides a number of motivations for corporate insurance but it does not provide a sufficiently clear role for the insurance contract to play in risk management that is different from that of other financial instruments![4] A positive risk management theory must provide some guidance in determining which financial instrument to use and when. This paper is an attempt at determining the role that insurance plays in a financial market setting and at specifying some circumstances that lead management to use insurance as well as other instruments in managing corporate risk.

The role that insurance plays in financial markets can be appreciated by distinguishing between risks. The finance paradigm categorizes the notion of risk as systematic or non-systematic, or equivalently, as diversifiable or non-diversifiable. One might also characterize the risks in the finance paradigm as generic and specific.[5] The insurance paradigm, however, classifies risks as either speculative or pure (Magee 1961; Bickelhaupt 1974). A speculative risk is characterized by a random payoff that may be either positive or negative; an investor may make a capital gain or a capital loss on most financial contracts and so the finance paradigm might be described as only classifying risks as speculative. A pure risk is characterized by a random payoff that is non-positive with probability one. The losses covered by insurance contracts are of this variety.

This analysis extends the scope of the existing literature on financial markets by introducing and exploring the notion of pure risks as well as speculative risks and by viewing insurance

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[1] The economics literature does show that risk averse agents have an incentive to purchase insurance from risk neutral agents, i.e., insurance companies, in a competitive insurance market but the analysis has not been replicated in a financial market setting. In fact, if it were and the usual definition of complete financial markets was employed then we could obtain another irrelevance result in which risk averse individuals would be indifferent between purchasing insurance and purchasing a portfolio of securities.

[2] See (MacMinn 1987). There are also other generalizations that show other financial instruments to be redundant. For example, see (MacMinn 1987).

[3] The current paradigm is the complete financial market model; it is not the CAPM.

[4] The literature in finance also provides a number of motivations for hedging but does not typically distinguish between hedging instruments or provide guidance on the use of the many hedging instruments available, e.g. see (Grossman 1975; Holthausen 1979; Feder, Just et al. 1980; Stulz 1984; Smith and Stulz 1985; Fabozzi 1988; Leuthold, Junkus et al. 1989; Campbell and Kracaw 1990; Ladd and Hanson 1991; Priovolos and Duncan 1991); also see (Froot, Scharfstein et al. 1993) and (Stulz 1996).

[5] See (Leland 1978; Satterthwaite 1981).

contracts as just one means of transferring risk in an integrated financial market setting. The liability system determines an initial allocation of pure risk bearing. The financial markets allow that allocation to be altered via trading among risk averse agents. Other things being equal,[6] investors should be able to diversify the pure corporate risks. This, in turn, should allow the firm's financial instruments to be valued. What is more, given the other things being equal assumption, this analysis yields a generalized version of the 1958 Modigliani-Miller theorem, which shows that the value of the insured firm equals that of the uninsured firm. This result follows because of a no arbitrage result that shows that the premiums on the insurance contracts must equal the value of the portfolios that investors would have to form to diversify the pure risks.

Once the other things being equal assumption is relaxed, it is possible to show that the insurance contract is preferred to the investors' diversification on personal accounts. While investors can hedge the corporate pure risks, that hedging behavior does not affect the corporation's probability of events such as financial distress. An appropriately structured insurance contract can change that probability of financial distress and so alter the incentives associated with the corporate investment and operating decisions. Therefore, the insurance contract can change decisions and corporate values by altering financial distress events. By reducing the risk of financial distress, the insurance contracts motivate corporate decisions that generate an efficient allocation of resources and risk bearing.

Since the current literature does not contain a financial market model that incorporates and uses pure and speculative risks, it has not been possible to address a number of questions that arise naturally in this setting. For example, are financial futures contracts and insurance contracts substitutes or complements? Both contracts are instruments that can be used to hedge risk. The futures contract is designed for speculative risks while the insurance contract is designed for pure risk. The analysis here shows that there are some cases in which the two contract types are complements rather than substitutes. The futures contract only affects the earnings distribution but the joint use of insurance and futures can reduce financial distress and so alleviate the adverse incentive problems. This is one step in the direct of integrating the notions of risk management in insurance and finance.

In the next section, the notions of pure and speculative risks are introduced in a financial market economy. A random process different from that of the speculative risks characterizes the pure risk. The analysis shows how investors or corporations can hedge these risks and how the pure risks are valued. In section three, the model is structured so that the firm makes financing, insurance, and futures decisions, and then subsequently makes a production decision. The analysis shows those cases in which insurance and futures can be used to change incentives and increase corporate value. Section four contains the concluding remarks.

Pure and Speculative Risks

Consider an economy operating between the dates t = 0 and 1, i.e., *now* and *then*, respectively. All decisions are made *now* while all payoffs on those decisions are received *then*. The financial market system will be assumed to be competitive and, in the absence of pure risks, will also be assumed to be complete. The economy is composed of real and fictitious agents. The real agents are the investors while the fictitious agents are the corporations. Investors make portfolio decisions on personal account to maximize expected utility subject to a budget constraint and all investors are assumed to be risk averse. The fictitious agents act in behalf of their principals, i.e., the investors who are shareholders.[7] The risks in this economy are characterized by states $\xi \in \Xi$, where Ξ is the set of states. In the standard complete financial market model, a state may be interpreted as an index of economic conditions and it is assumed that there are as many stock contracts as there are states of nature; each stock contract pays one dollar in a

[6]The 1958 Modigliani-Miller theorem at least implicitly assumes that capital structure is the only variable. The theorem does not allow for changes in either investment or operating decisions. Hence, although changing the capital structure may have implications for the probability of bankruptcy or financial distress which, in turn, can impact the investment or operating decisions, those implications are not explored. The other things being equal assumption here refers to the fixed investment and operating decision implicit in the 1958 Modigliani-Miller theorem.

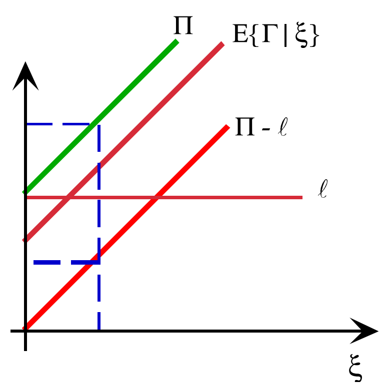
[7]This assumption is not necessary but it is made here for expediency. One could also assume that the corporation is managed by a real agent and then derive the objective function that the agent uses in making decisions for the corporation, e.g., see (MacMinn 1989; MacMinn and Page 1991).

particular state and zero otherwise. A corporate stock is then interpreted as a portfolio of these stock contracts. The corporate stock is a speculative risk.

In the economy constructed here the state space is expanded so that $(\xi, \zeta) \in \Xi \times Z$; ξ is interpreted as an index of economic conditions and $\Xi = [0, \omega]$ is the set of these index numbers. The state ζ represents an accident state and $Z = \{0, 1\}$ is the set of these states. The pure risk is a random variable $\Lambda: Z \rightarrow R$.[8] Let $\Lambda(0) = 0$ and $\Lambda(1) = \ell$. The corporate payoff is $\Gamma = \Pi - \Lambda$, where Π is the random corporate payoff without the pure risk.

This generalization of the financial model introduces a new valuation problem. Even if investors purchasing stock in the corporation know that a particular economic state will occur *then*, the corporate payoff is still uncertain until the accident state has been resolved. This is shown in the accompanying figure. The introduction of a pure risk causes incompleteness in an otherwise complete financial market system. If the corporation does not hedge the pure risk, risk averse investors will seek a means of hedging it. The construction of a hedge may be sketched as follows: To hedge the pure risk, consider the construction of a portfolio of put and call options. Let $E(\xi)$ be the exercise price on the options written against state ξ. Suppose an investor goes long in corporate stock and puts and short in calls. The payoff on a state ξ put option is $\max\{0, E(\xi) - \Gamma(\xi, \zeta)\}$ while the payoff on a state ξ call is $\max\{0, \Gamma(\xi, \zeta) - E(\xi)\}$. Such a portfolio of options allows the investor to hedge the pure risk since the payoff on the portfolio is $\Gamma + \max\{0, E - \Gamma\} - \max\{0, \Gamma - E\} = E$. If the exercise price on the puts and calls is set at the expected value of the corporate payoff given state ξ, i.e., $E(\xi) = E\{\Gamma \mid \xi\}$, for each ξ then Jensen's Inequality may be used to show that all risk averse investors prefer to hedge the pure risk.

Figure 1



This hedging behavior allows for a simple statement of the stock market value of the corporation *now*. Let $p(\xi)$ denote the price *now* of a stock that pays one dollar in state ξ and zero otherwise and suppose such a stock exists for each ξ. Let $\Psi(s)$ denote the sum of these stock prices over the set $\{\xi \in \Xi \mid \xi \leq s\}$. Since the hedged payoff on the corporate stock is $E\{\Gamma \mid \xi\} = \Pi(q, \xi) - E\Lambda$, it may be shown that the stock value of the uninsured firm is S^u where

$$\begin{aligned} S^u &= \int_\Xi E\{\Gamma \mid \xi\}\, d\Psi(\xi) \\ &= \int_\Xi \left(\Pi(q, \xi) - E\Lambda\right) d\Psi(\xi) \end{aligned} \quad (1)$$

[9]

[8]Any random variable may be interpreted as a function mapping index numbers into the real line. A speculative risk maps Ξ into the real line while a pure risk maps Z into the real line.

[9]This Riemann-Stieltjes form for the integral allows us to consider either a continuum of states or a finite number of states in Ξ.

The analysis will also show that if it is possible to fully insure the loss, then the stock market value of the insured firm is

$$S^i = -\, i + \int_\Xi \Pi(q, \xi)\, d\Psi(\xi) \tag{2}$$

A no arbitrage argument shows that the stock value of the uninsured firm equals that of the insured firm. Equivalently, the insurance premium must equal the present value of the expected loss. Therefore, the two representations of stock value are equal and establish a generalization of the 1958 Modigliani-Miller theorem (Modigliani and Miller 1958) since it shows that, *ceteris paribus*, the introduction of pure risk does not alter the earlier generalizations in the literature which showed that the value of the uninsured firm equals that of the insured firm.

Other hedging strategies exist. Consider the following hedging scheme that shows how the structure of an option portfolio compares to an insurance contract. Suppose the corporation constructs a portfolio of put options with exercise prices $\Pi(q, \xi)$ in each state ξ. The payoff on the corporation plus the payoff on the put portfolio is $\Gamma + \max\{0, \Pi - \Gamma\} = \Pi$. Hence, this portfolio of put options eliminates the pure risk and the value of the firm is its corporate value given no loss minus the value of the put option portfolio. Let S^p denote this value. Letting P denote the value of the put option portfolio, stock value may be expressed as

$$S^p = -\, P + \int_\Xi \Pi(q, \xi)\, d\Psi(\xi) \tag{3}$$

and a no arbitrage argument establishes the result that the value of the put portfolio equals the value of the insurance premium equals the present value of the expected loss, i.e., $P = i = p\, E\Lambda$, where p is the sum of the basis stock prices, or equivalently, the discount factor for a safe asset. Note that the put portfolio can be interpreted as an insurance contract since

$$\max\{0, \Pi - \Gamma\} = \text{Max}\{0, \Pi - (\Pi - \Lambda)\} = \max\{0, \Lambda\}. \tag{4}$$

Hence, the payoff on the put portfolio is zero if no loss occurs and the loss if it does occur. Also observe that (4) indicates that the contract could have simply been written on the accident states rather than the economic states. This would have allowed the firm to write one contract rather than one put for each economic state; it may also be noted that if investors must hedge then the number of options doubles when compared to the firm's put portfolio. Therefore, there is an efficiency gain due to the existence of insurance markets. Condition (4) also shows that the value of the insurance contract is equivalent to the value of the put option portfolio and so we have a way of determining the premium! If there is a deductible then the insurance takes the form $\max\{0, \Lambda - d\}$ where d is the deductible; the insurance premium becomes

$$i = \int_\Xi E(\Lambda - d)\, d\Psi = \int_\Xi \theta\, (\ell - d)\, d\Psi \tag{5}$$

This sketch of the analysis provides the base case. A relaxation of the *ceteris paribus* assumptions implicit in the 1958 Modigliani-Miller theorem will allow a characterization of the conditions under which insurance contracts play a positive role in alleviating conflict of interest problems and aligning incentives to achieve an efficient allocation of risks and resources.

Finance and Insurance

The finance and insurance decisions are modeled in this section. The firm is assumed to make a sequence of decisions.[10] The first set of decisions determines the corporate risk, or

[10] The model could be constructed in a multi-period format so that each set of decisions is made at a different date but that would required more complex notation without changing the results that are reported here.

equivalently, the contract set that the firm uses to raise capital and control risk. The firm can use any mix of debt, equity, and insurance contracts to raise capital and control the speculative and pure risks. The second set of decisions determines the corporate operations. The two sets of decisions are not independent. This analysis focuses on the interdependencies that can occur given a generic form of the risk shifting, equivalently, moral hazard problem that the firm faces in making operating decisions.

A risk shifting problem exists when it is possible for the firm to increase the stock value and decrease the bond value by increasing the risk of the corporate operations (Jensen and Meckling 1976; Mayers and Smith 1982; Jensen and Smith 1985). In the absence of a solution to the risk-shifting problem, the stockholders bear the agency cost. In a setting with no pure risks, (Green 1984) and (MacMinn 1989) showed that a properly structured convertible bond could be used to solve the problem. (MacMinn 1987) showed that an insurance contract could also be used to solve the risk-shifting problem. The analysis here differs from the earlier literature by allowing the pure and speculative risks to be generated by different random processes.

Here it is assumed that the two sets of decisions are made *now*. In the first step, the firm raises I dollars to invest through bond or equity issues and selects its insurance coverage. In the second step, the firm selects a production level q; that decision, like the others, is made prior to knowing the payoff $\Pi(q, \xi)$ and prior to knowing whether or not a loss occurs. The corporate payoff Π is defined as $\Pi(q, \xi) = P(\xi)\, q - c(q)$, where $P(\xi)$ is the random spot price *then* and c(q) is the cost function. Assume that the cost function is convex so that the corporate payoff is concave in the production level q. The structure of this corporate payoff is that of a competitive firm facing price uncertainty (Baron 1970; Sandmo 1971) and the payoff satisfies the Principle of Increasing Uncertainty, i.e., PIU, (Leland 1972). It has been shown (MacMinn and Holtmann 1983) that the PIU implies that after adjusting for a change in the expected payoff, this principle guarantees that an increase in the production increases risk in the Rothschild-Stiglitz sense, (Rothschild and Stiglitz 1970).

Basic Model

Suppose the firm does not use insurance to control risk. Then the equity payoff is $\max\{0, \Pi(q, \xi) - \Lambda - b\}$ and the stock market value is

$$S = \int_0^{\delta} (1-\theta)\,(\Pi - b)\, d\Psi + \int_{\delta}^{\omega} (\Pi - \theta d - b)\, d\Psi \tag{6}$$

where δ is the boundary of the bankruptcy event and is implicitly defined by the condition $\Pi(q, \delta) - b - d = 0$. The firm selects the production at this date to maximize current shareholder value. Now, consider the production decision of the levered firm. Let q^l denote the production decision that is implicitly defined by the condition

$$\begin{aligned}\frac{\partial S}{\partial q} &= \int_0^{\delta} (1-\theta)\, \frac{\partial \Pi}{\partial q}\, d\Psi + \int_{\delta}^{\omega} \frac{\partial \Pi}{\partial q}\, d\Psi \\ &= \int_0^{\omega} \frac{\partial \Pi}{\partial q}\, d\Psi - \theta \int_0^{\delta} \frac{\partial \Pi}{\partial q}\, d\Psi \\ &= 0\end{aligned} \tag{7}$$

Let q^u denote the optimal production decision of the firm with no default risk. The production decision q^u is implicitly defined by the condition

$$\int_0^{\omega} \frac{\partial \Pi}{\partial q}\, d\Psi = 0 \tag{8}$$

Observe that the production decision is greater with default risk, i.e., $q^l > q^u$. This is a variant of the risk-shifting problem. This result is summarized in the following theorem.

Theorem 1. If the principle of increasing uncertainty holds and there is a positive probability of insolvency then, *ceteris paribus*, the production decision of the levered firm is greater than that of the unlevered firm, i.e., $q^l > q^u$.

Next, consider how the production decision is affected by the financing and insurance decisions. The production decision will be denoted as q(b, d) if such a function exists. The next lemma establishes the existence of the function and its derivative properties.

Lemma 1: Let the second partial of the stock value with respect to the output be negative and let the marginal corporate payoff be negative at the boundary of the default event.[11] Then a function q(b, d) exists and is increasing in each of its arguments for all b and d such that the probability of default is positive.

Proof. By the Implicit Function Theorem, the derivatives are

$$\frac{\partial q}{\partial b} = -\frac{\dfrac{\partial^2 S}{\partial b \partial q}}{\dfrac{\partial^2 S}{\partial q^2}} = \frac{\theta\, p(\delta)\, \dfrac{\partial \Pi(q,\delta)}{\partial q}\, \dfrac{\partial \delta}{\partial b}}{\dfrac{\partial^2 S}{\partial q^2}} > 0 \tag{9}$$

$$\frac{\partial q}{\partial d} = -\frac{\dfrac{\partial^2 S}{\partial d \partial q}}{\dfrac{\partial^2 S}{\partial q^2}} = \frac{\theta\, p(\delta)\, \dfrac{\partial \Pi(q,\delta)}{\partial q}\, \dfrac{\partial \delta}{\partial d}}{\dfrac{\partial^2 S}{\partial q^2}} > 0 \tag{10}$$

The inequalities in (9) and (10) follow because δ is increasing in both b and d. QED

The lemma shows how the production decision is affected by the financing and insurance decisions in the previous stage.

The firm will make financing and insurance decisions in the first stage of the decision sequence knowing what impact those decisions have on the subsequent production decision. Hence, the firm will make those decisions to maximize $F \equiv V - I - i$, where i is the insurance premium

$$i = \int_0^\omega E(\Lambda - d)\, d\Psi = \int_0^\omega \theta\,(\ell - d)\, d\Psi$$

V is corporate value; V = B + S, where B represents the bond value. Note that the bond value is

$$B(b, d) = \int_0^\delta \big((1-\theta)b + \theta\,(\Pi(q,\xi) - d)\big)\, d\Psi + \int_\delta^\omega b\, d\Psi \tag{11}$$

Hence, the corporate value is

[11] It was not necessary to assume the negative marginal payoff at δ in models without the pure risk because that result was guaranteed by the PIU. Here it is not because stockholders do receive a payoff for states $\xi < \delta$ if no accident occurs. It is possible to motivate the condition by supposing a convex cost function. Then letting η be the state such that • Π/• q = 0 yields $P(\eta) = c'(q)$ as the implicit definition of η. The implicit definition of δ is $P(\delta) = (c(q) + b + d)/q$. Hence $\delta < \eta$ if $P(\delta) < P(\eta)$ or equivalently, $(c(q) + b + d)/q < c'(q)$. Roughly put, the marginal corporate payoff is negative at the boundary of the insolvency event if the average cost of production and finance is less than the marginal production cost.

$$V(b, d) = \int_0^{\delta} \left((1-\theta)\Pi + \theta\,(\Pi - d)\right) d\Psi + \int_{\delta}^{\omega} (\Pi - \theta d)\, d\Psi \tag{12}$$

The next lemma establishes F = V - I - i as the objective function.

Lemma 2. The objective function is F = V - I - i.
Proof: The manager makes the finance and insurance decisions to maximize current shareholder value subject to the constraint, i.e.,

$$\text{maximize } S^o$$
$$\text{subject to } S^n + D = I + i$$

Let the LaGrange function be $L = S^o + \lambda\,(S^n + D - I - I)$.

$$\frac{\partial L}{\partial n} = \frac{\partial S^o}{\partial n} + \lambda\,\frac{\partial S^n}{\partial n}$$

$$= -\frac{N}{(N+n)^2}\,S + \lambda\,\frac{(N+n)-n}{(N+n)^2}\,S$$

$$= 0$$

The lemma follows because this first order condition yields a LaGrange multiplier equal to one. QED

The objective function is expressed with the leverage b as an argument rather than the number of new shares and the leverage; once the leverage is determined the number of new shares is determined to raise the remaining amount required.

The following theorem characterizes the optimal deductible.

Theorem 2. *Ceteris paribus*, insuring is optimal if there is a positive probability of insolvency.
Proof. The capital structure and insurance decisions maximize F(b, d). The optimal deductible is implicitly defined by

$$\frac{\partial F}{\partial d} = \int_0^{\delta} \left((1-\theta)\frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial d} + \theta\left(\frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial d} - 1\right)\right) d\Psi + \int_{\delta}^{\omega} \left(\frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial d} - \theta\right) d\Psi + \int_0^{\omega} \theta\, d\Psi$$

$$= \int_0^{\delta} \left((1-\theta)\frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial d} + \theta\,\frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial d}\right) d\Psi + \int_{\delta}^{\omega} \frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial d}\, d\Psi \tag{13}$$

$$= 0$$

If the derivative is evaluated at q^I then

$$\frac{\partial F}{\partial d} = \frac{\partial q}{\partial d}\,\theta \int_0^{\delta} \frac{\partial \Pi}{\partial q}\, d\Psi < 0 \tag{14}$$

QED

This theorem shows that the firm has an incentive to insure in order to reduce default risk. The second equality in (13) follows because, with no change in production, the insurance decision yields a zero risk adjusted net present value. The change in production allows a change in the risk adjusted net present value. *Ceteris paribus*, the corporation has an incentive to reduce its deductible, equivalently, increase its insurance coverage, as long as the risk-shifting problem exists. Current shareholders receive the additional value created by reducing what would otherwise be an agency cost. If the firm eliminates default risk then it follows that the production choice is socially efficient.

While the theorem demonstrates an incentive to insure, it should be noted that other instruments could be used to achieve the same results. As in theorem two, differentiating the corporate objective with respect to b and evaluating the derivative at the optimal production level yields

$$\frac{\partial F}{\partial b} = \int_0^{\delta} \left((1-\theta)\frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial b} + \theta \frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial b} \right) d\Psi + \int_{\delta}^{\omega} \frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial b}\, d\Psi$$

$$= \frac{\partial q}{\partial b}\, \theta \int_0^{\delta} \frac{\partial \Pi}{\partial q}\, d\Psi \qquad (15)$$

$$< 0$$

Hence, *ceteris paribus*, it is optimal to reduce leverage if there is a positive probability of default.

By inspecting the derivatives in (14) and (15), it is apparent that the firm will be indifferent between increasing its insurance coverage and decreasing it leverage in this version of the model. The indifference is in terms of firm value. If the firm increases its leverage by a dollar and decreases its deductible by a dollar then it is moving in a direction v = (1, - 1) and the change in the value of the objective function in the direction v is D_vF where

$$D_vF = \frac{\partial F}{\partial b} - \frac{\partial F}{\partial d}$$

$$= \int_0^{\delta} \left((1-\theta)\frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial b} + \theta \frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial b} \right) d\Psi + \int_{\delta}^{\omega} \frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial b}\, d\Psi$$

$$- \int_0^{\delta} \left((1-\theta)\frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial d} + \theta \frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial d} \right) d\Psi - \int_{\delta}^{\omega} \frac{\partial \Pi}{\partial q}\frac{\partial q}{\partial d}\, d\Psi$$

$$= 0$$

There is, however, a linkage between the bond and insurance contracts. If the firm increases its leverage enough to make the probability of default positive then it must insure or lose value!

Corporate Tax

The tax model has traditionally been the most important in corporate finance. The corporate tax motivates the use of debt and helps explain the optimal use of that contract either in the small or in the large.

Suppose the tax liability of the corporation is $T = t \max\{0, \Pi - b - \Lambda\}$; this assumes that the principle and interest are deductible.[12] The equity payoff is $\Pi - b - \Lambda - T$ and so the stock value is

$$S = \int_0^{\delta}\big((1-\theta)(1-t)\big(\Pi(q,\xi) - b\big)\big)\, d\Psi + \int_{\delta}^{\omega}\big((1-\theta)(1-t)\big(\Pi(q,\xi) - b\big) + \theta\,(1-t)\big(\Pi(q,\xi) - b - d\big)\big)\, d\Psi \tag{16}$$

The corporate value becomes

$$V = \int_0^{\delta}\big((1-\theta)(1-t)\,\Pi + (1-\theta)\,tb + \theta\big(\Pi - d\big)\big)\, d\Psi + \int_{\delta}^{\omega}\big((1-t)\Pi + tb - \theta(1-t)d\big)\, d\Psi \tag{17}$$

Let the corporate objective be $F \equiv V - I - i$, as before, but where corporate value is now specified by (17).

The manager, acting in the interests of current shareholders, makes the finance and insurance decisions to maximize the objective function F. The first order condition for a bond issue is

$$\begin{aligned}\frac{\partial F}{\partial b} &= (1-\theta)\, t\int_0^{\delta} d\Psi + t\int_{\delta}^{\omega} d\Psi + \frac{\partial q}{\partial b}\left\{\int_0^{\delta}\left((1-\theta)(1-t)\,\frac{\partial \Pi}{\partial q} + \theta\frac{\partial \Pi}{\partial q}\right) d\Psi + \int_{\delta}^{\omega}(1-t)\frac{\partial \Pi}{\partial q}\, d\Psi\right\} \\ &= (1-\theta)\, t\int_0^{\delta} d\Psi + t\int_{\delta}^{\omega} d\Psi + \frac{\partial q}{\partial b}\left\{\int_0^{\delta}\theta\frac{\partial \Pi}{\partial q}\, d\Psi\right\}\end{aligned} \tag{18}$$

The second equality in (18) follows due to the stage two first order condition for an optimal output. The first two terms on the right hand side of (18) represent the marginal value of the debt tax shelter while the last term on the right hand side represents the marginal agency cost of the bond issue. Equation (18) implies the result that the firm issues bonds and pushes the bond issue to the point at which the marginal value of the tax shelter equals the marginal cost of the agency problem. Hence, *ceteris paribus*, (18) implies a risky debt issue.

The manager also makes an insurance decision to maximize current shareholder value. The first order condition is

[12] The assumption is only made to simplify the analysis and make the models here approximately the same.

$$\frac{\partial F}{\partial d} = -\theta \int_0^{\delta} d\Psi - (1-t)\,\theta \int_{\delta}^{\omega} d\Psi + \theta \int_0^{\omega} d\Psi$$

$$+ \frac{\partial q}{\partial d} \left\{ \int_0^{\delta} \left((1-\theta)(1-t)\frac{\partial \Pi}{\partial q} + \theta \frac{\partial \Pi}{\partial q} \right) d\Psi + \int_{\delta}^{\omega} (1-t)\frac{\partial \Pi}{\partial q}\, d\Psi \right\} \qquad (19)$$

$$= \theta\, t \int_{\delta}^{\omega} d\Psi + \frac{\partial q}{\partial d} \left\{ \int_0^{\delta} \theta \frac{\partial \Pi}{\partial q}\, d\Psi \right\}$$

The second equality follows due to the stage two first order condition for an optimal output. The first term on the right hand side of (19) represents the marginal value of the tax shelter while the second term on the right hand side represents the marginal agency cost. (19) implies the result that the firm increases its deductible, equivalently, reduces its insurance to the point at which the marginal value of the tax shelter equals the marginal agency cost. (19) does not yield a conclusion like the bond issue because setting the deductible to zero does not eliminate the default risk; the contrary is more nearly true.

Despite the limitations in interpreting the first order condition in (19), it is possible to demonstrate a demand for insurance in this version of the model. It is possible for the firm to increase its leverage with a bond issue and counter the increase in the agency cost by simultaneously increasing its insurance coverage. A one to one trade-off in the size of the bond issue and the size of the deductible suffices to eliminate the agency cost at the margin and to increase the value of the tax shelter. Hence, there is a tax driven demand for insurance. The result is summarized in the following theorem.

Theorem 3. The corporate tax suffices to generate a demand for insurance.
Sketch of Proof. Suppose that for every dollar increase in leverage, the firm reduces the deductible by a dollar. Then the firm can generate an increase in value. Letting v = (1, - 1) and D_vF denote the derivative of the objective function in the direction v, observe that

$$D_vF(b,d) = \frac{\partial F}{\partial b} - \frac{\partial F}{\partial d}$$

$$= (1-\theta)\, t \int_0^{\omega} d\Psi \qquad (20)$$

$$> 0$$

QED

Theorem three shows a strong motivation to insure.[13] The loss or deductible represents a risky tax shelter while the bond represents a certain tax shelter; the theorem shows that it is optimal to replace a risky with a safe tax shelter since the latter is more valuable. It should also be noted that the theorem demonstrates a demand for full insurance since the derivative in the direction of more insurance is always positive. The theorem shows that value can be increased as long as there are uninsured losses.

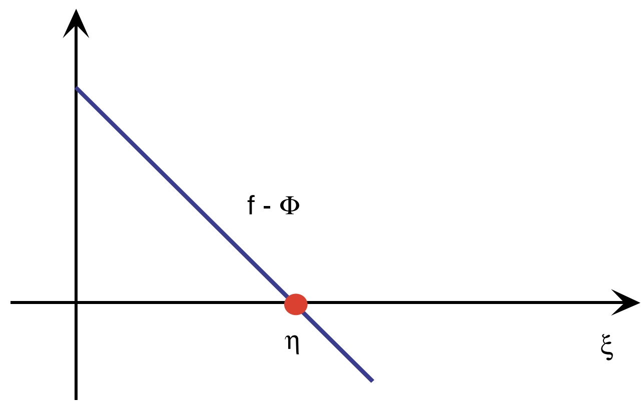
[13] Of course it should be recalled that the principal and interest are being deducted here. In a setting in which only interest on debt is deductible a similar result obtains but also requires the interest rate, i.e., coupon interest on the issue, to be sufficiently large relative to the probability of the loss.

Forwards and Futures

The insurance contract is not the only contract that the firm can use in managing corporate risk. It is also possible to use forward and futures contracts. A positive theory of risk management must provide some rationale for the use of each type of instrument. To the extent that both insurance and futures contracts can be used to hedge risk, it is natural to assert that the two contracts are substitutes. If so then it is only necessary to use one or the other. It is possible to frame an argument that says that the firm will be indifferent to the use of either contractual form.[14] The analysis here shows that when the insurance mechanism does not yield undiluted incentives, other mechanisms can complement the risk transfer capabilities of the insurance contract. The analysis shows that a hedging scheme for the corporation exists that provides the right incentives and increases value.

Consider a futures contract. Let Φ denote the unit payoff on a stock index and let f denote the forward price. Suppose the unit payoff is increasing in state ξ. The payoff on the futures position is $\varphi\,(f - \Phi(\xi))$, where φ is the position taken by the firm in futures. The payoff on the futures contract is shown in figure two. The payoff depicted is sometimes referred to as a short position in the futures contract.

Figure 2



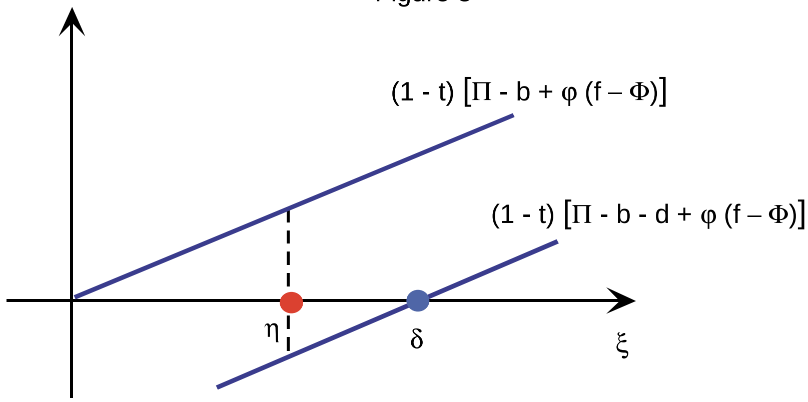
One possible comparison that is instructive is the difference in corporate value between an insured firm and a hedged firm. In the insured case, without loss of generality, suppose that full insurance is selected. In the hedged case, also suppose that the firm selects a fully hedged position. If the probability of default is greater than a capital gain on the futures contracts then the claim here is that the value of the insured firm is greater than that of the hedged firm; the hedge, in this case, is that in financial futures. If the claim holds then it provides a strong motivation for the use of insurance and, at the same time, it shows that futures and insurance contracts may be viewed as complements in some cases.

Suppose, for the moment that the firm either hedges with financial futures or with insurance. Claim that under certain circumstances the value of the insured firm exceeds the value of the uninsured firm. The payoffs for the firm hedging in financial futures are shown in the following figure. The equity payoffs are the maxima of zero and the payoffs indicated in the figure. The bond payoff is Π - d + $\varphi\,(f - \Phi)$ if $\xi < \delta$ with probability θ and b if $\xi > \delta$ with probability θ.[15] The payoffs rotate about their values at η and so the probability of insolvency increases in the event of an accident if

[14] Of course, the 1958 Modigliani-Miller theorem also suggests that the firm will not be able to increase value by using either contract but theorem two in the previous section shows that once the ceteris paribus assumptions implicit in the MM 58 theorem are relaxed, it is possible to increase value by altering the firm's contract set.

[15] The value for the deductible is ℓ here if the hedged firm is uninsured.

Figure 3



$$P\{\Phi < f\} < P\{(1-t)[\Pi - b - d + \varphi(f - \Phi)] < 0\}$$

or equivalently, if $\eta < \delta$. Hedging in this case increases the probability of default in the event of an accident. It follows that the payoff in the event of an accident goes to zero as the firm increases its hedge in financial futures. In this case the corporate payoff of the hedged firm becomes $(1 – t)(\Pi + \varphi\ (f - \Phi))$ in the event of no accident and zero otherwise.[16] Given full insurance, the corporate payoff of the insured firm becomes $(1 – t)(\Pi + \varphi\ (f - \Phi))$ in either the accident or no accident events. Letting V^h and V^i denote the hedged and insured corporate values, respectively, it follows that the difference in corporate values is

$$V^i - V^h = \theta \int_0^{\omega} (1-t)[\Pi + \varphi(f - \Phi)] d\Psi$$

$$= \theta(1-t) \int_0^{\omega} \Pi \, d\Psi$$

$$= \theta V$$

$$> 0$$

Hence the insured value does exceed the hedged value. Insurance is preferred if the difference in corporate value exceeds the insurance premium paid now, equivalently

$$V^i – V^h = \theta\, V > i = \theta\, p\, \ell, \tag{21}$$

where p denotes the sum of the basis stock prices or equivalently the discount factor for a safe asset. The inequality in (21) follows if the corporate value exceeds the present value of the loss. This is a rather innocuous condition but it yields a powerful result! It shows that the financial risk management afforded by futures is not sufficient to maximize current shareholder value. It also shows that current shareholder value cannot be maximized without the use of insurance. In this case, the financial futures and insurance contracts are not substitutes. The result here shows that the firm does not have an incentive to use futures contracts in isolation since a full hedge does not capture all the value that can be captured with insurance or, more generally, a combination of futures and insurance. The firm can generally do better by insuring and, as the subsequent

16 The corporate payoff is the stockholder and bondholder payoff.

analysis shows, continuing to hedge with financial futures if there is any additional value that can be preserved.

Next, consider the conditions that lead to a joint use of futures and insurance in managing risk. The corporate payoff is (1 - t) max{0, (Π - b - Λ + max{0, Λ - d} + φ (f - Φ))}. Theorem three showed that the firm has an incentive to fully insure its property risk and equation (18) showed that the firm has an incentive to issue risky bonds. Now suppose the firm fully insures the pure risk but that a positive probability of insolvency remains. With a deductible of zero, the corporate payoff function becomes (1 - t) max{0, (Π - b + φ (f - Φ))}. The payoffs are shown in figure three; the payoff rotates clockwise through the intersection at state η shown in the figure. The objective of this section is to show that the firm can increase current shareholder value by hedging in futures contracts.

The positive probability of default even with full insurance leaves the firm with a risk-shifting problem *now* and so the shareholders must bear the remaining agency cost of the debt issue. The stock market value of a fully insured firm is

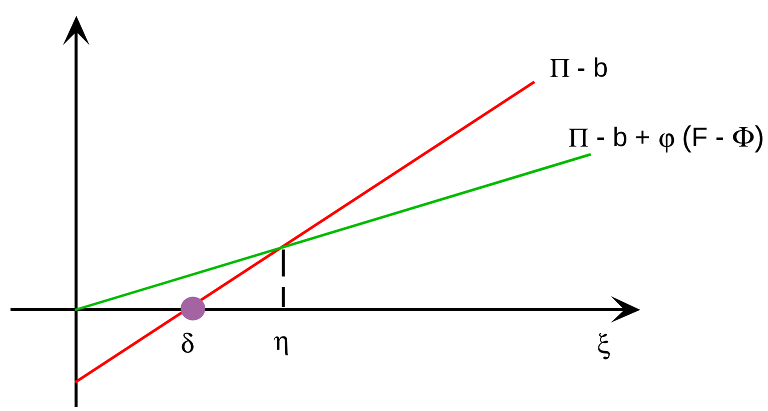
$$S = \int_{\delta}^{\omega} (1 - t) \left(\Pi(q, \xi) - b + \varphi \left(f - \Phi \right) \right) d\Psi \tag{22}$$

where the boundary δ of the insolvency event is implicitly defined by the condition Π(q, δ) - b + φ (f - Φ(δ)) = 0; the condition for an optimal production decision is

$$\frac{\partial S}{\partial q} = (1 - t) \int_{\delta}^{\omega} \frac{\partial \Pi(q, \xi)}{\partial q} d\Psi = 0 \tag{23}$$

This first order condition yields a production decision greater than that of an insured unlevered firm, i.e., $q^l > q^u$ similar to theorem one. The next lemma shows that hedging in futures reduces the output decision.

Figure 4



Lemma 3. Let the state η be implicitly defined by Φ(η) = f, and let η > δ for the unhedged firm. The production decision is decreasing in the size of the hedge.

Proof. The optimal output is a decreasing function of the size of the hedge since (23) implies

$$\frac{\partial q}{\partial \varphi} = -\frac{\dfrac{\partial^2 S}{\partial q \partial \varphi}}{\dfrac{\partial^2 S}{\partial q^2}}$$

$$= -\frac{-(1-t)\,\dfrac{\partial \Pi(q,\delta)}{\partial q}\,p(\delta)\,\dfrac{\partial \delta}{\partial \varphi}}{\dfrac{\partial^2 S}{\partial q^2}} \qquad (24)$$

$$< 0$$

The inequality in (24) follows since $\partial\Pi/\partial q$ is negative at δ by the PIU and $\partial\delta/\partial\varphi$ is negative for $\delta < \eta$. QED

Lemma three shows that the futures contracts can decrease the output decision. It follows because the corporate hedge reduces the agency costs of the leverage; hedging moves the firm closer to a socially efficient production decision, or equivalently, one that maximizes value for all corporate stakeholders.

While the lemma shows that the production level is reduced, it does not establish the incentive to hedge. The manager makes the finance, insurance, and futures decisions to maximize corporate value. The bond and corporate values are

$$B = \int_0^{\delta} \left(\Pi(q,\xi) + \varphi\,(f - \Phi)\right) d\Psi + \int_{\delta}^{\omega} b\, d\Psi \qquad (25)$$

and

$$V = B + S$$

$$= \int_0^{\delta} \left(\Pi(q,\xi) + \varphi\,(f - \Phi)\right) d\Psi \qquad (26)$$

$$+ \int_{\delta}^{\omega} \left[(1-t)\left(\Pi(q,\xi) + \varphi\,(f - \Phi)\right) + t\,b\right] d\Psi$$

respectively.

The following theorem shows that forward contracts can be used in conjunction with insurance contracts to increase value and control risk.

Theorem 4. If $P\{\Pi < 0\} > 0$ and $P\{\Pi < 0\} < P\{\Phi < f\}$, then futures contracts form part of the insured firm's optimal contract set.

Proof. It suffices to show that the derivative of F with respect to φ is positive at $\varphi = 0$.

$$\frac{\partial F}{\partial \varphi} = \int_0^{\delta} \left(\frac{\partial \Pi}{\partial q} \frac{\partial q}{\partial \varphi} + \left(f - \Phi\right) \right) d\Psi$$

$$+ \int_{\delta}^{\omega} (1-t) \left(\frac{\partial \Pi}{\partial q} \frac{\partial q}{\partial \varphi} + \left(f - \Phi\right) \right) d\Psi \qquad (27)$$

Evaluating corporate value at the optimal production level yields

$$\frac{\partial F}{\partial \varphi} = \frac{\partial q}{\partial \varphi} \int_0^{\delta} \frac{\partial \Pi}{\partial q} d\Psi + t \int_0^{\delta} \left(f - \Phi\right) d\Psi$$

$$> 0 \qquad (28)$$

The inequality in (28) follows because the risk adjusted net present value of the forward contract is zero, i.e.,

$$\int_0^{\omega} \left(f - \Phi\right) d\Psi = 0 \, ,$$

because the optimal output decreases with the size of the hedge φ, and because the PIU yields a negative marginal payoff in the event of insolvency, i.e., $\partial \Pi / \partial q < 0$ for all $\xi \in (0, \delta]$. QED

This theorem demonstrates a case in which the joint use of insurance and financial futures is optimal. By insuring the property risk, the firm increases the value of its debt tax shelter; just insuring, however, does not reduce or eliminate the risk-shifting problem. The firm can continue to increase value by hedging with financial futures; this will reduce the risk-shifting problem and so create value that can be captured by the current shareholders.

Concluding Remarks

Although the finance and insurance disciplines are related, the language in each of the literatures is often quite different. The theory of insurance markets is often considered in isolation, i.e., without reference to other financial markets. The theory of financial markets is also typically considered without pure risks. This research is an attempt to provide a synthesis of finance and insurance. While the analysis shows that investors or corporations can hedge the pure risks, it also shows that only the hedging on corporate account changes incentives and increases value. The insurance contract provides one of the most efficient means of hedging pure risks because the contract is structured for just that purpose. The corporation can purchase one insurance contract and achieve the alteration of incentives that it would take a whole portfolio of options to accomplish.

This analysis is also an attempt at clarifying the role of risk management. A risk management theory should provide not only a menu of the tools that management has available for hedging risk but also some indication when tools should be used. The analysis here takes a few steps in that direction. Theorem two shows that, other things being equal, the firm should insure whenever there is a positive probability of financial distress; the analysis also shows that the risk of financial distress can be controlled by unlevering the firm. Theorem three introduces a corporate tax and shows that it is optimal to insure and lever the firm because that will maximize the value of the tax shelter; hence, theorem three also establishes an optimal capital structure result. Finally, theorem four introduces futures contracts and shows that if the probability of financial distress is less than the probability of a capital loss on the futures portfolio then it is optimal to hedge with futures contracts in addition to the insurance contracts.

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