Competition and Compensation

by

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I. Introduction

The financial theory of the firm is based on the belief that managers act in the interest of their corporate shareholders. Indeed, corporate management has a fiduciary responsibility to act in the shareholder' interest. Another seemingly popular belief is that paying corporate managers with stock options aligns their interests with those of their shareholders. It is a simple matter, however, to show that stock options do not, *ceteris paribus*, align incentives. This observation raises the following question: Why are so many corporate managers paid with stock options?[1] The purpose of this analysis is to provide one answer to that question.

The stock option is a call option that gives its owner the right to purchase a specified number of shares at an exercise price that is established when the contract is written. If the exercise price is less than the stock price when the option vests then the option is said to be in the money; otherwise it is out of the money. If there is a positive probability that the stock options will be out of the money then the interests of manager and shareholders are not aligned because they do not share the same downside risk. In maximizing the value of the stock options rather than the current shareholder value, managers have an incentive to increase risk that has been documented by (DeFusco, Johnson et al. 1990). The analysis here provides the theoretical foundation for that increase in risk and shows that, *ceteris paribus*, the increase in risk yields a smaller corporate value.

The economy considered here consists of competitive financial markets and labor markets and an imperfectly competitive product market. The corporate board of directors hires a manager in the competitive labor market and determines the provisions

[1]Stock options are important because nearly 90% of Fortune 500 companies offer potentially lucrative incentives to their executives and those incentives include stock options. See Joann S. Lubin, "The American Advantage," *Wall Street Journal*, Wednesday, April 17, 1991, R4.

of the manager's compensation contract in the first stage of the analysis. Then the manager makes the production decisions in the second stage of the analysis. Since the board must form rational expectations about the behavior of the manager based on the compensations contract provisions, these stages are considered in reverse order as in all dynamic programming problems. The analysis shows the conditions under which the corporate board of directors will rationally choose to compensate the manager with stock options.

The next section sketches the imperfectly competitive product market in a financial market setting, i.e., the second stage of the problem. It demonstrates how the manager makes decisions on behalf of the corporation and how those decisions are affected by changes in the compensation contract. In section III, the first stage of the analysis is presented. The analysis shows how the board makes the compensation decision and the conditions under which that compensation contract includes stock options. Section IV provides concluding remarks.

II . The Financial Market Model

Consider an economy composed of product and financial markets in which all agents interacting in the markets are risk averse. Suppose two corporations operate in an imperfectly competitive product market and finance their operations in a competitive financial market. Suppose the product market is characterized by a Cournot duopoly.

The following notiation will be used in the construction of the model:

ξ	state of nature
$\Omega = [0, \omega]$	set of states
q_f	output of firm f = 1, 2
q	output vector (q_1, q_2)

$c_f(q_f)$	cost function for corporation f
$h(q_1 + q_2, \xi)$	demand function
$\Pi_f(q, \xi)$	payoff function for corporation f
$p(\xi)$	basis stock price
$P(\xi)$	sum of basis stock prices; $P(\xi) = \int_0^\xi p(t)$
n_f	number of stock options
N_f	number of common stock shares *now*
a_f	manager's stake given exercise; $n_f / (n_f + N_f)$
e_f	exercise price *then*
W_f	warrant value
V_f	corporate value

Given a Cournot duopoly, the payoff of corporation one may be characterized as

$$\Pi_1(q_1, q_2, \xi) = h(q_1 + q_2, \xi)\, q_1 - c_1(q_1) \tag{1}$$

where h is the random market demand function. The payoff of corporation two is characterized similarly. Suppose the random demand function satisfies the Principle of Increasing Uncertainty (PIU) (Leland 1972; MacMinn and Holtmann 1983) so that $\partial h/\partial\xi > 0$ and $\partial^2 h/\partial q_f \partial\xi > 0$ for f = 1, 2. It follows that the payoff functions also satisfy the PIU since

$$\frac{\partial \Pi_1}{\partial \xi} = \frac{\partial h}{\partial \xi} q_1 > 0 \text{ and } \frac{\partial^2 \Pi_1}{\partial q_1 \partial \xi} = \frac{\partial h}{\partial \xi} + \frac{\partial^2 h}{\partial q_1 \partial \xi} q_1 > 0 \tag{2}$$

These, of course, are simply the conditions that revenue and marginal revenue are increasing in state.

Each corporation also operates in the financial market. It is supposed here that all investors are risk averse and transfer money from now to then or then to now by selecting portfolios in the financial market. It is assumed here that the financial market is complete. In such a setting it is possible to define stock that pays one dollar *then* in state ξ and zero otherwise (Arrow 1963). Call these contracts the basis stock. Then it is apparent that all contracts can be expressed as a portfolio of basis stock. Let $p(\xi)$ denote the price of a basis stock *now* and let $P(\xi)$ denote the sum of the basis stock prices from zero to ξ. Then the corporate value V_f of firm f is the risk adjusted present value of the payoff, i.e.,

$$V_f = \int_{\Omega} \Pi(q, \xi)\, dP(\xi) \tag{3}$$

Next, suppose the manager of corporation f is compensated with a stock option package. Suppose that each option gives the manager the right to purchase one share of stock then at an exercise price of e_f dollars. Let N_f represent the number of share of common stock outstanding now and let n_f represent the number of new shares that must be issued then if the manager exercises the n_f options in the stock option package. The payoff on the stock option package is

$$\max\left\{0,\, n_f \left(\frac{\Pi_f + e_f\, n_f}{N_f + n_f} - e_f\right)\right\} = \max\left\{0,\, a_f \left(\Pi_f - e_f\, N_f\right)\right\} \tag{4}$$

Then the value of the stock option package is W_f, where

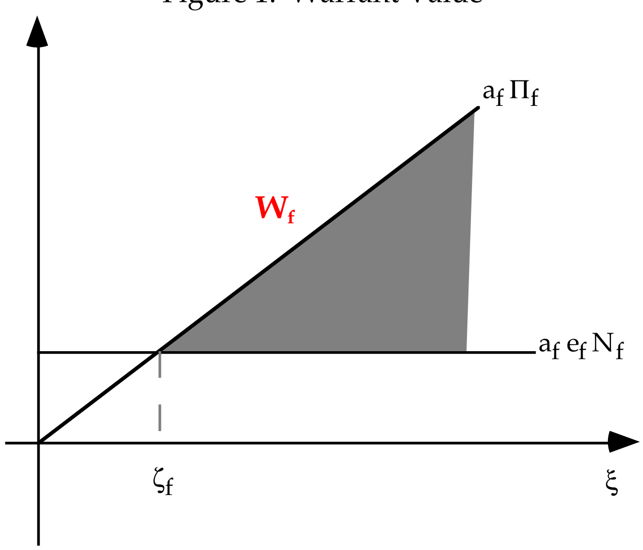
$$W_f(e_f, n_f) = \int_{\Omega} \max\left\{0, n_f\left(\frac{\Pi_f + e_f\, n_f}{N_f + n_f} - e_f\right)\right\} dP \qquad (5)$$

The options will be referred to as "in the money" if the payoff in (4) is positive and "out of the money" otherwise. Let ζ_f denote the boundary of the in the money event, or equivalently, exercise event. If the out of the money, or equivalently, the no exercise event is not empty, then ζ_f is implicitly defined by the condition $\Pi_f(q, \zeta_f) - e_f\, N_f = 0$. It should be observed that the boundary of the exercise event is a function of q and e_f. The value of the stock option package may be equivalently expressed as

$$W_f(a_f, e_f) = \int_{\zeta_f}^{\omega} a_f\left[\Pi_f - e_f N_f\right] dP \qquad (6)$$

where a_f is defined as the manager's stake in the corporation upon exercise, i.e., $a_f \equiv n_f/(N_f + n_f)$. This warrant value is proportional to the area shown in the figure.

Figure 1: Warrant Value



Finally, observe that the stock market value of the corporation is $S_f = V_f - W_f$.

Assume that the board of directors in each corporation has set its executive compensation and that each manager determines an optimal output in the product market. A simple Fisher separation result shows that a manager making decisions in his or her own self-interest will make decisions on corporate account to maximize the value of the compensation package. Hence, in this case the manager makes a production decision to maximize the value of the stock option package.[2] In a Cournot duopoly each corporation takes the output of the other as given and in equilibrium this assumption is satisfied. The first and second order conditions are

$$\frac{\partial W_f}{\partial q_f} = \int_{\zeta_f}^{\omega} a_f \frac{\partial \Pi_f}{\partial q_f}\, dP = 0\,,\ f = 1, 2 \tag{7}$$

$$\begin{aligned} \frac{\partial^2 W_f}{\partial^2 q_f} &= -\ p(\zeta_f)\ a_f \frac{\partial \Pi_f}{\partial q_f} \frac{\partial \zeta_f}{\partial q_f} \\ &\quad + \int_{\zeta_f}^{\omega} a_f \frac{\partial^2 \Pi_f}{\partial^2 q_f}\, dP < 0 \end{aligned} \tag{8}$$

for f = 1, 2. The simulataneous solution of the conditions in (7) is the Cournot duopoly solution. A Cournot solution also requires the following two conditions:

$$\frac{\partial^2 W_f}{\partial q_f\, \partial q_g} < 0,\ \text{for } f, g = 1, 2 \tag{9}$$

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[2]Notethat the manager may be given a salary now and then and that this will not have an impact on the results stated here as long as this other component of the compensation package does not depend on the manager's decisions on corporate account.

$$\frac{\partial^2 W_1}{\partial q_1\,\partial q_1}\,\frac{\partial^2 W_2}{\partial q_2\,\partial q_2} - \frac{\partial^2 W_1}{\partial q_1\,\partial q_2}\,\frac{\partial^2 W_2}{\partial q_2\,\partial q_1} > 0 \tag{10}$$

Condition (9) guarantees that the reaction functions are decreasing and condition (10) guarantees that the equilibrium is stable. These conditions are demonstrated in the figure.

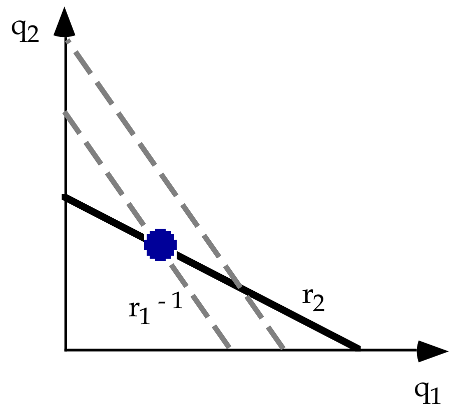

Figure 2: Reaction Functions

Alternatively, one may derive the reaction functions. Note that

$$\frac{\partial W_1(q, e_1)}{\partial q_1} = \int_{\zeta_1}^{\omega} a_1\,\frac{\partial \Pi_1}{\partial q_1}\,dP \tag{11}$$

and if the second order condition is satisfied then it follows by the Implicit Function Theorem that there exists a differentiable function r_1 such that $q_1 = r_1(e_1, q_2)$, equation (11) equals zero[3] and

[3]The reaction function must satisfy the condition

$$\frac{\partial W_1(e_1,\ r_1(e_1,\ q_2),\ q_2)}{\partial q_1} = 0$$

Setting the differential equal to zero and rearranging yields

$$dq_1 = -\frac{\dfrac{\partial^2 W_1}{\partial q_1\,\partial e_1}}{\dfrac{\partial^2 W_1}{\partial^2 q_1}}\,de_1 - \frac{\dfrac{\partial^2 W_1}{\partial q_1\,\partial q_2}}{\dfrac{\partial^2 W_1}{\partial^2 q_1}}\,dq_2$$

and the partial derivatives of the reaction function are based on this result.

$$\frac{\partial r_1}{\partial q_2} = - \frac{\dfrac{\partial^2 W_1}{\partial q_2\, \partial q_1}}{\dfrac{\partial^2 W_1}{\partial^2 q_1}} \tag{12}$$

Then duopolist one decreases output as duopolist two increases output if the numerator in (12) is negative. The sign of the numerator is not obvious as the following derivative shows

$$\frac{\partial^2 W_1}{\partial q_1\, \partial q_2} = - \, p(\zeta_1)\, a_1 \frac{\partial \Pi_1}{\partial q_1} \frac{\partial \zeta_1}{\partial q_2} + \int_{\zeta_1}^{\omega} a_1 \frac{\partial^2 \Pi_1}{\partial q_1\, \partial q_2}\, dP$$

One would expect the cross partial of the payoff function to be negative since that would say that the marginal revenue of firm one is decreasing in the output of firm two. Hence, the integral would be negative. In addition, the marginal payoff, i.e., $\partial \Pi(q, \zeta_1)/\partial q_1$, is negative by the PIU. The boundary of the exercise event for the firm one manager, however, is increasing in the output of firm two; equivalently, the probability of exercise decreases as the output of the rival firm increases. Hence, the first term is positive and the sign is ambiguous. This shows the necessity for the assumption made in Equation (9) .

Similarly, observe how the reaction function of this duopolist changes with the exercise price e_1. Note that

$$\frac{\partial r_1}{\partial e_1} = - \frac{\dfrac{\partial^2 W_1}{\partial q_1\, \partial e_1}}{\dfrac{\partial^2 W_1}{\partial^2 q_1}} \tag{13}$$

Hence, the reaction function is increasing in e_1 if and only if the numerator is positive. Note that

$$\frac{\partial^2 W_1}{\partial q_1\, \partial e_1} = -\, p(\zeta_1)\, a_1 \frac{\partial \Pi_1}{\partial q_1} \frac{\partial \zeta_1}{\partial e_1} > 0 \tag{14}$$

since $\partial\Pi_1/q_1$ is negative at ζ_1 by the PIU and ζ_1 is an increasing function of e_1 since implicit differentiation of $\Pi_1(q, \zeta_1)$ - e_1 N_1 = 0 yields

$$\frac{\partial \Pi_1}{\partial \zeta_1} \frac{\partial \zeta_1}{\partial e_1} - N_1 = 0$$

or equivalently

$$\frac{\partial \zeta_1}{\partial e_1} = \frac{N_1}{\frac{\partial \Pi_1}{\partial \zeta_1}} > 0$$

These results combine to show that the reaction function of the duopolist is increasing in exercise price, i.e., $\partial r_1/\partial e_1 > 0$, as long as the warrant issue has some positive probability of being out of the money. This result is demonstrated in the figure. Of course, it is also apparent that the reaction function is not increasing in the exercise price as long as the probability of being out of the money is zero.

Consider the comparative statics associated with an increase in the exercise price of corporation one. The differential of the system of equations in (7) may be stated as

$$\frac{\partial^2 W_1}{\partial q_1^2}\, dq_1 + \frac{\partial^2 W_1}{\partial q_1\, \partial q_2}\, dq_2 + \frac{\partial^2 W_1}{\partial q_1\, \partial e_1}\, de_1 = 0$$

$$\frac{\partial^2 W_2}{\partial q_2\,\partial q_1}\,dq_1 + \frac{\partial^2 W_2}{\partial q_2^2}\,dq_2 + \frac{\partial^2 W_2}{\partial q_2\,\partial e_1}\,de_1 = 0$$

or in matrix form as

$$\begin{bmatrix} \dfrac{\partial^2 W_1}{\partial q_1^2} & \dfrac{\partial^2 W_1}{\partial q_1\,\partial q_2} \\ \dfrac{\partial^2 W_2}{\partial q_2\,\partial q_1} & \dfrac{\partial^2 W_2}{\partial q_2^2} \end{bmatrix} \begin{pmatrix} dq_1 \\ dq_2 \end{pmatrix} = -\begin{pmatrix} \dfrac{\partial^2 W_1}{\partial q_1\,\partial e_1} \\ \dfrac{\partial^2 W_2}{\partial q_2\,\partial e_1} \end{pmatrix} de_1$$

Let

$$\left|H\right| \equiv \begin{vmatrix} \dfrac{\partial^2 W_1}{\partial q_1^2} & \dfrac{\partial^2 W_1}{\partial q_1\,\partial q_2} \\ \dfrac{\partial^2 W_2}{\partial q_2\,\partial q_1} & \dfrac{\partial^2 W_2}{\partial q_2^2} \end{vmatrix} > 0 \tag{15}$$

The inequality in (15) follows by (10), i.e., by the stability condition. Then by Cramer's Rule

$$\frac{dq_1}{de_1} = \frac{\begin{vmatrix} -\dfrac{\partial^2 W_1}{\partial q_1\,\partial e_1} & \dfrac{\partial^2 W_1}{\partial q_1\,\partial q_2} \\ -\dfrac{\partial^2 W_2}{\partial q_2\,\partial e_1} & \dfrac{\partial^2 W_2}{\partial q_2^2} \end{vmatrix}}{\left|H\right|} = -\frac{\dfrac{\partial^2 W_1}{\partial q_1\,\partial e_1}\,\dfrac{\partial^2 W_2}{\partial q_2^2}}{\left|H\right|} > 0 \tag{16}$$

and

$$\frac{dq_2}{de_1} = \frac{\begin{vmatrix} \frac{\partial^2 W_1}{\partial q_1^2} & -\frac{\partial^2 W_1}{\partial q_1 \, \partial e_1} \\ \frac{\partial^2 W_2}{\partial q_2 \, \partial q_1} & -\frac{\partial^2 W_2}{\partial q_2 \, \partial e_1} \end{vmatrix}}{\left|H\right|} = \frac{\frac{\partial^2 W_1}{\partial q_2 \, \partial q_1} \frac{\partial^2 W_1}{\partial q_1 \, \partial e_1}}{\left|H\right|} < 0 \qquad (17)$$

The inequality in (16) follows by (8), (10), and (14). Similarly, the inequality in (17) follows by (9), (10), and (14).

III . The Compensation Decision

Next, consider the compensation decisions. The results in the previous section show that executive compensation can be used as a strategic commitment device. Suppose the board of directors of corporation f = 1, 2, makes the compensation decision when hiring the manager in a competitive labor market. Also suppose that the board members have equity stakes in the corporation. The board then has an incentive to maximize current shareholder value in making decisions on corporate account.[4]

Consider the sequence of decisions. At date zero, the board of directors selects a compensation scheme and hires a manager. The compensation scheme must provide the manager with the risk adjusted dollar equivalent of the competitive level of compensation determined in the labor market. Then the corporate managers selects a production level. The production level cannot be observed by shareholders but they are rational and so understand the nature of the incentives management faces as well as the nature of the competition in the duopoly product market. Hence, for compensation

[4]It is not necessary to assume that all board members have an equity stake in the corporation. The rational pursuit of self-interest shows that those who don't will be indifferent to decisions on corporate account. Hence, decisions made to maximize current shareholder value will be unanymous.

package (a, e), the stockholders can anticipate the production levels. The current shareholder value for corporation f = 1, 2, is

$$S_f = \int_{\Omega} \left[\Pi_f - \max\left\{0, a_f \left(\Pi_f - e_f\, N_f\right)\right\}\right] dP$$

$$= \int_0^{\zeta_f} \Pi_f \; dP - a_f \int_{\zeta_f}^{\omega} \left(\Pi_f - e_f\, N_f\right) dP \tag{18}$$

Given a board with interests aligned to the shareholders and a competitive managerial labor market, the board's constrained maximization problem may be stated as

$$\text{maximize } S_f\,(a_f, e_f)$$
$$\text{subject to } W_f\,(a_f, e_f) = c \tag{19}$$

where c represents the cash equivalent of the equilibrium compensation in the labor market.

Now consider the board's compensation decision. The constrained maximization problem in (19) may be equivalently expressed in Lagrange form as

$$\text{maximize } L(a_f, e_f, \lambda)$$

where $L = S_f + \lambda\,(W_f - c)$. The conditions for a maximum are as follows:

$$\frac{\partial L}{\partial a_f} = \frac{\partial S_f}{\partial a_f} + \lambda\,\frac{\partial W_f}{\partial a_f} = 0 \tag{20}$$

$$\frac{\partial L}{\partial e_f} = \frac{\partial S_f}{\partial e_f} + \lambda \frac{\partial W_f}{\partial e_f} = 0 \tag{21}$$

Direct calculation shows that $\frac{\partial S_f}{\partial a_f} = -\frac{\partial W_f}{\partial a_f}$ and so condition (20) yields $\lambda = 1$. Recall that $V_f = S_f + W_f$ and so by (20), condition (21) may be rewritten as

$$\frac{\partial S_f}{\partial e_f} + \frac{\partial W_f}{\partial e_f} = \frac{\partial V_f}{\partial e_f} = 0 \tag{22}$$

Note that

$$\frac{\partial V_f}{\partial e_f} = \frac{\partial q_f}{\partial e_f} \int_0^{\omega} \frac{\partial \Pi_f}{\partial q_f}\, dP + \frac{\partial q_g}{\partial e_f} \int_0^{\omega} \frac{\partial \Pi_f}{\partial q_g}\, dP \tag{23}$$

The first term on the right hand side of (23) is negative. To see this recall that $\partial q_f / \partial e_f > 0$ by (16) and note that

$$\int_0^{\omega} \frac{\partial \Pi_f}{\partial q_f}\, dP = \int_0^{\zeta_f} \frac{\partial \Pi_f}{\partial q_f}\, dP + \int_{\zeta_f}^{\omega} \frac{\partial \Pi_f}{\partial q_f}\, dP = \int_0^{\zeta_f} \frac{\partial \Pi_f}{\partial q_f}\, dP < 0 \tag{24}$$

The second equality in (24) follows by the first order condition (7) and the inequality follows because the PIU yields $\partial \Pi_f / \partial q_f < 0$ for all states $\xi \in (0, \zeta_f)$. The second term on the right hand side of (23), however, is positive because the payoff of corporation f is a decreasing function of the output of corporation g, i.e.,

$$\frac{\partial \Pi_f}{\partial q_g} = \frac{\partial h}{\partial q_g} q_f < 0$$

and because $\partial q_g/\partial e_f < 0$ by (17). The two effects shown in (23) may be easily interpreted. First, the negative term is a negative of the marginal agency cost of the stock option scheme. The stock option package provides the manager with an incentive to produce beyond the level that would be selected by a stockholder. *Ceteris paribus*, this reduces the value of the corporation. Hence, there is an agency cost associated with a stock option compensation package. Second, the positive term is the marginal benefit due to the competitor's reduced output, i.e., the gains due to the competitive advantage associated with this commitment mechanism. Although the particular exercise price is not clear this analysis does show that the exercise price must be set so that there is a positive probability that the options are not in the money. To see this simply suppose that the exercise price is the largest price such that the probability of exercise is one. At that exercise price the boundary state ζ_f is zero and increasing in e_f. At the largest exercise price that makes the probability of exercise one, the manager effectively has a stock grant and so chooses to maximize corporate value. Hence the first term on the right hand side of (23) disappears, leaving a positive derivative. It follows that the board of directors can increase corporate value by increasing the exercise price.

It should also be observed that the positive term on the RHS of (23) depends on the nature of the imperfect competition in the product market. If the product market is perfectly competitive then the positive term disappears and so the strategic benefits of stock options disappear leaving only the marginal agency cost. Hence, the board of directors for a firm operating in a competitive product market has an incentive to compensate a manager with stock rather than stock options. A similar statement can be made for monopoly.

IV. Concluding Remarks

References

Arrow, K. J. (1963). "The Role of Securities in the Optimal Allocation of Risk Bearing." Review of Economic Studies.

DeFusco, R. A., R. R. Johnson, et al. (1990). "The Effect of Executive Stock Option Plans on Stockholders and Bondholders." Journal of Finance **45**(2): 617-27.

Leland, H. (1972). "Theory of the Firm Facing Uncertain Demand." American Economic Review **62**: 278-91.

MacMinn, R. D. and A. Holtmann (1983). "Technological Uncertainty and the Theory of the Firm." Southern Economic Journal **50**: 120-36.