Hedging Brevity and Longevity Risk with Mortality-based Securities

by

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As populations grow and change, markets integrate and medical science advances, the character and structure of the risks faced by governments, corporations and individuals also change. The threat of SARS in 2003 and avian flu in 2004 have provided reminders that governments and life insurers face correlated risks on a large scale in events such as pandemics.

In 2004, Swiss Re introduced a mortality based security designed to hedge excessive mortality changes for its life book of business. The concern was apparently brevity risk, i.e., the risk of premature death. Brevity risk can be managed with the standard tools as long as there are no correlated mortality surprises. Such would be the case with a recurrence of the Spanish flu or more generally with the occurrence of a new avian flu. The potential for pandemics introduces correlated risks on a large scale and so the potential for mortality surprises. The brevity risk due to a pandemic is similar to the property risk associated with catastrophic events such as earthquakes and hurricanes and the security used to hedge the risk is similar to a CAT bond (Dubinsky and Laster 2003).

To date there have been no similar introductions of mortality based securities to hedge excessive mortality changes for annuity books of business; such a mortality based security would, of course, be designed to cover excessive mortality changes in the opposite direction. The concern here would be longevity risk, i.e., the risk of living too long. Mortality improvements are being reported; there has been acceleration in the mortality improvements at older ages in Sweden (Wilmoth, Deegan et al. 2000). There has also been some evidence that genetics plays a major role in the ability to survive to extremely old ages and hence that genetic research may yield insights into how to slow the aging process (Strauss 2001). The mortality improvements do yield correlated risks for insurers with life annuity books. To the extent that the improvements can be predicted accurately over the horizon of the life annuities, the longevity risk can be managed by insurers with the standard tools. Life annuities, however, have tails that are quite long and so although the mortality improvements may seem less surprising, the correlated risks are just as problematic.

Survivor bonds (Blake and Burrows 2001) have been suggested as an effective means of managing longevity risk. The survivor bond is essentially a reverse tontine; the bond pays a coupon that is proportional to the number of survivors in a cohort. A basis risk problem might remain depending on how the cohort is structured. An instrument similar to that issued by Swiss Re for brevity risk could also be structured for longevity risk. The Swiss Re instrument is in the money if the mortality rate becomes too large but one could also write a security that would be in the money if the mortality rate became too small.

The goal here is to analyze the potential usefulness of mortality based securities in hedging risk. An organization with books of life or annuity business will be constructed. The organization may be a government institution, reinsurer or insurer. That organization will be structured so that it faces brevity or longevity risk in addition to other standard forms of risk such as credit and interest rate risk. For simplicity, the analysis begins with a reinsurer that faces just brevity risk in addition to the standard capital market risks. A reinsurer facing a capital constraint may find a mortality based security to be a natural risk management tool. Once the potential value creation of a mortality security is clear for the reinsurer, the analysis will turn to the construction of an insurer with both the life and annuity books of business; such an insurer faces both brevity and longevity risk. The insurer has a number of risk management tools available. The annuity and life books can be structured to provide a natural hedge; the insurer can reinsure one or both books of business or turn to the capital markets to hedge one or both risks with mortality securities.

The literature on Alternative Risk Transfer (ART) provides explanations of how securitization of catastrophic exposures can create value. Various papers have identified the trade-offs involved in the design of optimal risk management programs integrating traditional (re)insurance and ART (Doherty 1997; Doherty 1997; Doherty 1997; Doherty 1997; Froot 1997; Froot 1999; Croson and Kunreuther 2000). On the one hand, securitization of insurance risk offers certain advantages over traditional reinsurance arrangements, such as the potential to substantially reduce moral hazard, credit risk and transaction costs. On the other hand, possible improvements typically come at a cost: For instance, the introduction of an index, such as a population's average life expectancy, as the trigger for coverage provided by a security issue reduces or eliminates moral hazard and may decrease transaction costs. However, it also creates basis risk if the index does not perfectly represent the individual risk to be hedged.

This work will consider the trade-offs an insurer or reinsurer faces in optimizing a hedging strategy for its life insurance or annuity exposure. The primary interest here is in the trade-off between the credit risk and the basis risk associated with an index or indemnity based security given the moral hazard problem faced in underwriting.

Model

Consider an organization in a competitive economy operating between the dates t = 0 and 1 (MacMinn 1987). The dates t = 0 and 1 are subsequently referred to as *now* and *then*, respectively. Decisions are made *now* and payoffs on those decisions are received *then*. The economy is composed of organizations and risk averse investors. Investors make portfolio decisions on personal account to maximize expected utility subject to a budget constraint. The organization will initially take the corporate form and will be assumed to act on behalf of its principals, i.e., the investors who are shareholders.[1]

The insurance company considered will face the standard capital market risks such as interest rate and credit risks but will also face annuity or life risk. The premium income will be generated *now* and invested in a capital market portfolio. The composition of the organization's portfolio will be determined endogenously and will be shown to depend on the risk management choices made. The losses on the books of business occur *then* and depend on the state of nature revealed. The losses on the annuity and life risks are negatively correlated; this is the feature that generates the condition for a natural hedge. The following partially summarizes the notation used in the development of the model:

[1] The assumption is only for convenience. The corporate objective function can be derived; for example, see MacMinn, R. (2004). The Fisher Model and Corporate Finance. Taipei, forthcoming..

ω	state of nature
$\Omega = [0, \zeta]$	set of states
$p(\omega)$	basis stock price *now*
$P(\omega)$	sum of basis stock prices $\varepsilon \le \omega$; $P(\omega) = \int_0^\omega p(\varepsilon)\,d\varepsilon$
$\Gamma_j(\omega)$ [2]	premium income *then* on book of business j = A, L
a_j	effort spent in composing the book of business measured in dollars, j = A, L
A	(a_A, a_L)
$L_j(a_j, \omega)$	loss on book of business j = A, L
$\Pi_j(a_j, \omega)$	payoff on book of business, i.e., $\Pi_j(a_j, \omega) = \Gamma_j(1 + R(\omega)) - L_j(a_j, \omega) - a_j$
$\Pi(a, \omega)$	corporate payoff, i.e., $\Pi = \Pi_A + \Pi_L$
$I_j(\omega)$	mortality index for j = A, L
i_j	exercise price for mortality security j = A, L
S	stock value

Suppose the financial markets are competitive. In the absence of any insurance linked security, the stock market value of the (re)insurer may be expressed as the value of its books of business as follows:

$$S(a) = \int_\Omega \max\{0, \Pi(a, \omega)\}\, dP(\omega) \tag{1}$$

[2] As the economy improves in state so does the premium income *then* since the premium income is invested.

Consider the reinsurer. The reinsurer has the payoff $\Pi(\omega)$ in the absence of the mortality based security. The reinsurer may create a mortality based security for its life book by forming a special purpose entity (SPE) similar to that for a CAT bond; the essence of the security from the perspective of the insurer, however, is the creation of an option that yields an indexed payoff of $I_L(\omega)$ dollars in state ω for losses on its life book in excess of a trigger amount i_L; equivalently, the security pays $\max\{0, I_L(\omega) - i_L\}$. Similarly, the insurer may create a mortality based security for its annuity book by forming an SPE; the essence of the security from the perspective of the insurer, however, is the creation of an option that yields an indexed payoff of $I_A(\omega)$ dollars in state ω for losses on its annuity book in excess of a trigger amount i_A; equivalently, the security pays $\max\{0, i_A - I_A(\omega)\}$.

In an economy such as this the 1958 Modigliani-Miller theorem (Modigliani and Miller 1958) will hold and so mortality based securities will not, *ceteris paribus*, increase the stock value of the (re)insurer. Once the *ceteris paribus* assumption is relaxed we will investigate the incentive effects of the mortality based securities and ultimately the impact on value. Additionally, the natural hedge between the reinsurer's life and annuity books of business may be investigated.

Few papers so far have addressed these trade-offs analytically: Doherty and Mahul (Doherty and Mahul 2001) and Doherty and Richter (Doherty and Richter 2002), investigate the interaction of moral hazard and basis risk, when insurance can be used to cover the basis risk of an index linked transaction. Nell and Richter (Nell and Richter 2004) study the trade-off between the implicit transaction cost incurred by a reinsurer's risk aversion and the basis risk of a CAT bond.

Cummins and Mahul (Cummins and Mahul 2000) consider an insurance product that is subject to default risk as well as basis risk,[3] as the insurer's payment is tied to an exogenous index. The interaction between these two factors is also analysed by Richter (Richter 2003) albeit with two different instruments: Insurance, on the one hand, is subject to default risk but can be used to generate a perfect hedge. Risk securitization, on the other hand, comes without default risk but incurs basis risk.

The existing literature on the theory of demand for reinsurance versus ART is exclusively concerned with innovative instruments used in the property and casualty area, e.g., CAT bonds. The analysis here focuses on life and annuity exposures and extends the previous work by incorporating default, credit, and basis risks in addition to moral hazard in the model.

Brevity Risk

From the (re)insurer's perspective the life book of business exposes the corporation to the risk that an insured's life is briefer than expected and so we refer to this as brevity risk. The reinsurer may also be exposed to insolvency risk and credit risk in the financial markets; the insolvency risk introduces the judgment proof problem with the associated incentive problems. The ability to securitize some risk in capital markets introduces a moral hazard

[3] The recently published version of (Cummins and Mahul 2004), however, does not include the basis risk.

problem in the underwriting operation. In this section we concentrate on the life book of business and the associated risks.

First, consider the value of the reinsurer with and without the mortality based security; we'll refer to the mortality based security more generally as an insurance linked security (ILS). The unhedged reinsurer, i.e., without an ILS, has the value

$$\begin{aligned} S^u(a) &= \int_\Omega \max\{0, \Pi(a,\omega)\}\, dP(\omega) \\ &= \int_\delta^\zeta \Pi(a,\omega)\, dP(\omega) \end{aligned} \tag{2}$$

where δ is the boundary of the insolvency event and is implicitly defined by the condition $\Pi(a,\delta) = 0$.[4] Observe that the reinsurer selects the underwriting effort to maximize the current shareholder value. The first order condition is

$$\frac{dS^u}{da} = \int_\delta^\zeta D_1\Pi\left(a^u,\omega\right) dP(\omega) = 0 \tag{3}$$ [5]

Equation (3) implicitly defines the optimal underwriting activity a^u. The underwriting effort by the reinsurer is assumed to reduce the brevity risk. This assumption is formalized in the following:

Assumption 1. The reinsurer's payoff $\Pi(a,\omega)$ satisfies the principle of decreasing uncertainty (PDU) and the PDU is defined by the following derivative properties:

$$\frac{\partial \Pi}{\partial \omega} > 0 \text{ and } \frac{\partial^2 \Pi}{\partial \omega \partial a} < 0 \tag{4}$$

After compensating for the change in the mean, the PDU provides a decrease in the risk of the payoff in the Rothschild-Stiglitz sense (Rothschild and Stiglitz 1970; MacMinn and Holtmann 1983).

Next, consider the second order condition. Observe that

$$\frac{d^2S^u}{da^2} = \int_\delta^\zeta D_{11}\Pi\left(a^u,\omega\right) dP(\omega) - D_1\Pi\left(a^u,\delta\right) p(\delta)\frac{d\delta}{da} < 0 \tag{5}$$

The concavity of Π or equivalently the convexity of L suffices to make the first term on the right hand side of (5) negative. The PDU suffices to show that $D_1\Pi(a^u,\delta) > 0$ and

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[4] To simplify notation, the subscript L for the life book of business will not be used in the remainder of this section.

[5] $D_1\Pi$ is standard notation for the partial derivative of the function Π with respect to its first argument.

$$\frac{d\delta}{da} = -\frac{D_1\Pi(a^u,\delta)}{D_2\Pi(a^u,\delta)} < 0 \tag{6}$$

Hence, the second order condition holds if the second term on the RHS of (5) is less that the first. It may also be noted that the second order condition reduces to just the first term in the absence of insolvency risk and so the concavity of the payoff suffices to show that the condition holds. We will assume that the second order condition is satisfied in the remaining analysis.

It is useful to compare the underwriting decisions of the firms with and without insolvency risk. Recall that Shavell has described the situation in which an insurer does not possess the resources to cover all losses with certainty as the judgment proof problem. The insurer facing the judgment proof problem does not have the incentive to select the socially efficient level of care as noted in the following claim and proof. Let a^e denote the level of care or equivalently the underwriting choice made by the reinsurer with no insolvency risk.[6]

Claim: The level of care selected by the reinsurer is greater in the absence of insolvency risk, i.e., $a^e > a^u$

Proof. In the absence of insolvency risk the value is S^e where

$$S^e(a) = \int_0^\zeta \Pi(a,\omega)\, dP(\omega) \tag{7}$$

and the first order condition for a socially optimal level of care is

$$\frac{dS^e}{da} = \int_0^\zeta D_1\Pi(a^e,\omega)\, dP(\omega) = 0 \tag{8}$$

Hence, the claim follows by noting that

$$\begin{aligned} \left.\frac{dS^e}{da} - \frac{dS^u}{da}\right|_{a=a^u} &= \int_0^\zeta D_1\Pi(a^u,\omega)\, dP(\omega) - \int_\delta^\zeta D_1\Pi(a^u,\omega)\, dP(\omega) \\ &= \int_0^\delta D_1\Pi(a^u,\omega)\, dP(\omega) \qquad (9) \\ &> 0 \end{aligned}$$

[6] The socially efficient care is that level that maximizes the value for all stakeholders in the enterprise and so can also be described in situations with insolvency risk as well. Equation (7) would still apply.

follows by the PDU since $D_1\Pi(a^u,\delta)$ is positive and $D_{12}\Pi < 0$ yields $D_1\Pi(a^u,\omega) > 0$ for all $\omega \le \delta$. QED

It may also be noted that the social optimum noted in equation (8) is, with appropriate discounting, equivalent to the optimum noted in the literature by (Shavell 1986; Kahan 1989; MacMinn 2002). The social optimum is the level of care such that the present value of the marginal benefit equals that of the marginal cost, as seen in the following rewrite of equation (8)

$$\begin{aligned} \frac{dS^e}{da} &= \int_0^\zeta \left(-D_1L(a^e,\omega) - 1\right) dP(\omega) \\ &= -\int_0^\zeta D_1L(a^e,\omega)\, dP(\omega) - \int_0^\zeta dP(\omega) \qquad (10) \\ &= 0 \end{aligned}$$

The first term on the RHS of the second equality is the marginal benefit or equivalently the present value of the marginal loss reduction and the second term is the marginal cost or equivalently the present value of the last dollar spent on care.

Next, consider the introduction of an insurance linked security. The case considered here is that of a mortality based bond issued by an SPE. The instrument is designed to pay the reinsurer in the event of a mortality surprise, e.g., if the mortality rate is 130% or more of what had been projected. Such an instrument may be constructed with an indemnity or index trigger. In the indemnity case the payoff from the perspective of the reinsurer would be an option payoff like $\max\{0, L(a,\omega) - i\}$ where i is the strike price. In the index case the payoff from the perspective of the corporation would be $\max\{0, I(\omega) - i\}$ where $I(\omega)$ is the loss index.

Indemnity trigger

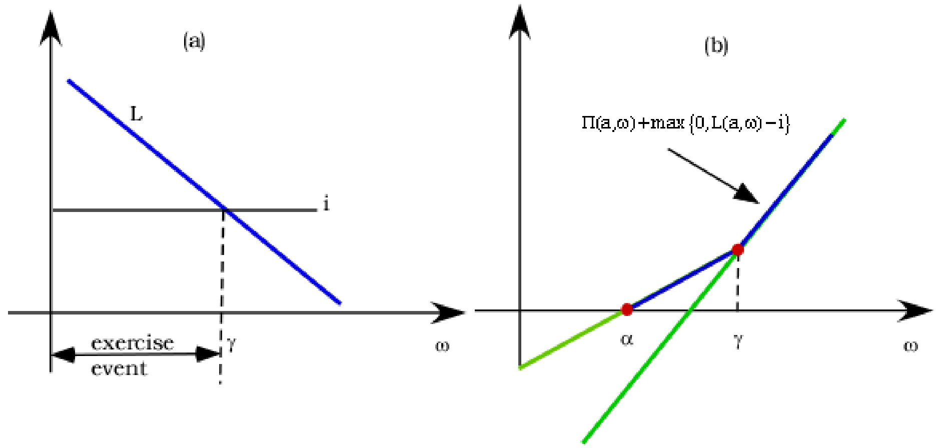
The indemnity trigger case of an insurance linked security costs C^m dollars now where C^m is the call option price for the coverage. Then

$$\begin{aligned} C^m(a,i) &= \int_\Omega \max\{0, L(a,\omega) - i\}\, dP \\ &= \int_0^\gamma \left(L(a,\omega) - i\right) dP \end{aligned} \qquad (11)$$

where γ is the boundary of the option's in the money event as shown in figure one.[7]

[7] The payoffs in the figures will be represented as linear due to the author's limited drawing ability; the analysis does not depend on the linear functions represented in the figures.

Figure 1



The stock value *then* of the corporation with this ILS is

$$
\begin{aligned}
S^m(a,i) &= \int_\Omega \max\left\{0, \Pi(a,\omega) + \max\{0, L(a,\omega) - i\}\right\} dP \\
&= \int_\alpha^\zeta \left(\Pi(a,\omega) + \max\{0, L(a,\omega) - i\}\right) dP
\end{aligned}
\tag{12}
$$

where α is the boundary of the insolvency event as shown in figure 1(b). Finally the current shareholder value is S^{mo}

$$
\begin{aligned}
S^{mo}(a,i) &= -C^m(a,i) + S^m(a,i) \\
&= -\int_0^\gamma \left(L(a,\omega) - i\right) dP + \int_\alpha^\zeta \left(\Pi(a,\omega) + \max\{0, L(a,\omega) - i\}\right) dP \\
&= -\int_0^\alpha \left(L(a,\omega) - i\right) dP + \int_\alpha^\zeta \Pi(a,\omega)\, dP
\end{aligned}
\tag{13}
$$

Let S^{uo} denote the current shareholder value in the unhedged case with no insolvency risk. It follows that if the probability of insolvency is zero so that α is zero then (13) becomes

$$
\begin{aligned}
S^{mo}(a,i) &= -C^m(a,i) + S^m(a,i) \\
&= -\int_0^\gamma \left(L(a,\omega) - i\right) dP + \int_0^\zeta \left(\Pi(a,\omega) + \max\{0, L(a,\omega) - i\}\right) dP \\
&= \int_0^\zeta \Pi(a,\omega)\, dP \\
&= S^{uo}(a)
\end{aligned}
\tag{14}
$$

and no value is added by the ILS. This is confirmation of the generalized version of the 1958 Modigliani-Miller theorem. It is achieved with the usual *ceteris paribus* assumption; in this case no change in underwriting care due to the ILS is allowed in making the comparison and that assumption will be changed in the subsequent analysis. It may also be noted that the current shareholder value S^{mo} diminishes relative to the unhedged current shareholder value S^{uo} given insolvency risk; this is not a real diminution of value but rather a redistribution of value from shareholders to policyholders.

Incentive effects of the indemnity trigger

The ILS with an indemnity trigger will have an impact on incentive to take care in underwriting. The indemnity trigger has the effect of full loss coverage in some states and that in turn impacts the underwriting choice; equivalently, a well known moral hazard problem (Shavell 1979) occurs with this form of the ILS. The relationship between the option coverage and the underwriting care will be specified in the function $a^m(i)$ where i is the exercise price of the option. The function a^m is implicitly defined by the first order condition for (12).

$$\begin{aligned}\frac{\partial S^m}{\partial a} &= \int_\alpha^\zeta \frac{\partial \Pi}{\partial a} dP + \int_\alpha^\gamma \frac{\partial L}{\partial a} dP \\ &= \int_\alpha^\zeta \left(-\frac{\partial L}{\partial a} - 1\right) dP + \int_\alpha^\gamma \frac{\partial L}{\partial a} dP \\ &= 0\end{aligned} \tag{15}$$

Suppose the second order condition for a maximum holds. Then the function $a^m(i)$ exists and its derivative is

$$\frac{da^m}{di} = -\frac{\dfrac{\partial^2 S^m}{\partial a\, \partial i}}{\dfrac{\partial^2 S^m}{\partial^2 a}} \geq 0 \tag{16}$$

if the numerator is non-negative. Observe that

$$\begin{aligned}\frac{\partial^2 S^m}{\partial a\, \partial i} &= \frac{\partial}{\partial i}\left(\int_\alpha^\zeta \left(-\frac{\partial L}{\partial a} - 1\right) dP + \int_\alpha^\gamma \frac{\partial L}{\partial a} dP\right) \\ &= -\left(-\frac{\partial L}{\partial a} - 1\right) p(\alpha) \frac{\partial \alpha}{\partial i} - \frac{\partial L}{\partial a} p(\alpha) \frac{\partial \alpha}{\partial i} + \frac{\partial L}{\partial a} p(\gamma) \frac{\partial \gamma}{\partial i} \\ &= p(\alpha) \frac{\partial \alpha}{\partial i} + \frac{\partial L}{\partial a} p(\gamma) \frac{\partial \gamma}{\partial i} \\ &> 0\end{aligned} \tag{17}$$

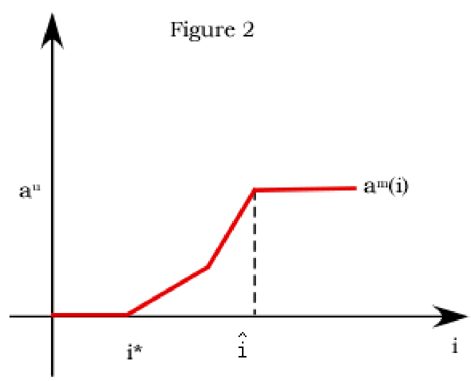
The inequality in (17) follows because α is increasing in i, γ is decreasing in i and, of course L is decreasing in a.[8] Therefore, the inequality in (16) is a strict inequality and $a^m(i)$ is increasing in i.

Figure one and equation (12) are appropriate for some exercise prices but there are two more generic cases that must also be included. First, there is an exercise price $\hat{i}$ sufficiently large that α equals δ; for larger i the boundary of the insolvency event remains at δ. For $i \geq \hat{i}$ the stock value is that provided in equation (2) and so $a^m(i)$ is a constant on the interval $i \geq \hat{i}$. Second, there is an exercise price i^* such that α equals zero and so i* eliminates insolvency risk; for $i \leq i^*$ the stock value is

$$\begin{aligned} S^m &= \int_0^{\gamma}(\Pi + L - i)\,dP + \int_{\gamma}^{\zeta}\Pi\,dP \\ &= \int_0^{\gamma}(\Gamma - a - i)\,dP + \int_{\gamma}^{\zeta}(\Gamma - L - a)\,dP \end{aligned} \quad (18)$$

and again it is apparent that $a^m(i)$ is a constant on the interval $i \leq i^*$; i* may but need not be as small as zero[9] but at i = 0 the care decision clearly goes to zero as shown in figure two. The results are collected in figure two. Note that $a^m(i)$ is non-decreasing and this is confirmation of a moral hazard problem since an increase in the strike price i is equivalent to less loss coverage and that generates more care in underwriting but more loss coverage generates less care. The figure also suggests the indemnity trigger ILS cannot provide the incentive for adequate care in underwriting since the maximum effort is that for the unhedged case.

Figure 2



[8] Note that α is implicitly defined by the condition $\Pi(a,\alpha) + (L(a,\alpha) - i) = 0$ or equivalently by $\Gamma(\alpha) - a - i = 0$ and so

$$\frac{\partial\alpha}{\partial i} = \frac{1}{\Gamma'(\alpha)} > 0\,.$$

Similarly, γ is implicitly defined by the condition $L(a,\gamma) - i = 0$ and so

$$\frac{\partial\gamma}{\partial i} = \frac{1}{\frac{\partial L}{\partial\gamma}} < 0\,.$$

[9] This claim may be justified by direct calculation since γ increases without limit.

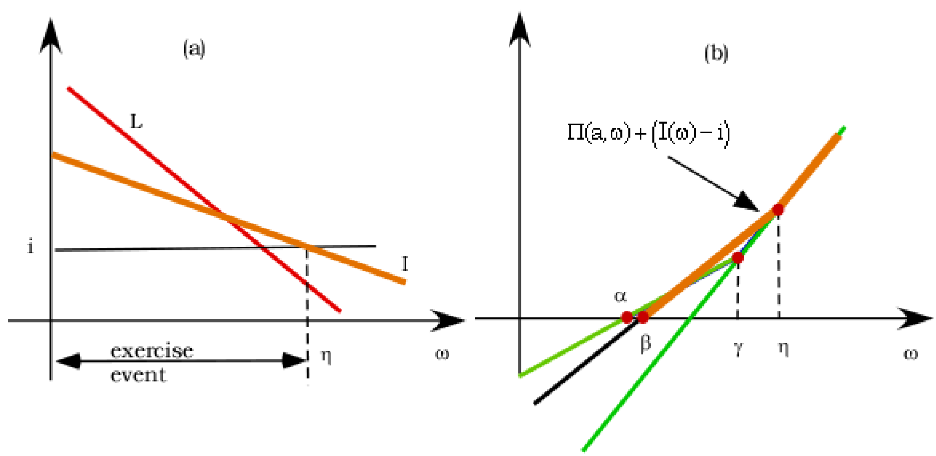
Index trigger

The index trigger case of an insurance linked security costs C^b dollars now where C^b is the call option price for the coverage. Then

$$\begin{aligned} C^b(i) &= \int_\Omega \max\{0, I(\omega) - i\} \, dP \\ &= \int_0^\eta \left(I(\omega) - i\right) dP \end{aligned} \tag{19}$$

where η is the boundary of the in the money or equivalently the exercise event for this option as shown in figure three.

Figure 3



The stock value of the corporation with this ILS is

$$\begin{aligned} S^b(a,i) &= \int_\Omega \max\{0, \Pi(a,\omega) + \max\{0, I(\omega) - i\}\} \, dP \\ &= \int_\beta^\zeta \left(\Pi(a,\omega) + \max\{0, I(\omega) - i\}\right) dP \end{aligned} \tag{20}$$

where β is the boundary of the insolvency event as shown in figure 3(b). The current shareholder value in this case is

$$
\begin{aligned}
S^{bo}(a,i) &= -C^b(i) + S^b(a,i) \\
&= -\int_0^{\eta}\big(I(\omega)-i\big)dP + \int_{\beta}^{\zeta}\big(\Pi(a,\omega) + \max\{0, I(\omega)-i\}\big)dP \\
&= -\int_0^{\eta}\big(I(\omega)-i\big)dP + \int_{\beta}^{\eta}\big(\Gamma(\omega) - a + \big(I(\omega) - L(a,\omega)\big) - i\big)dP \\
&\quad + \int_{\eta}^{\zeta}\big(\Gamma(\omega) - a - L(a,\omega)\big)dP
\end{aligned} \tag{21}
$$

Note that the second term on the RHS of the third equality includes the basis risk (I – L). The connection with the generalized 1958 Modigliani-Miller theorem can be made here as in the last case when the insolvency risk is zero.

Incentive effects of the index trigger

The ILS with an index trigger will have an impact on incentive to take care in underwriting. Unlike the indemnity trigger, this instrument does generate a moral hazard problem but it does involve basis risk. The relationship between the option coverage and the underwriting care will be specified in the function $a^b(i)$ where i is the exercise price of the option. The function a^b is implicitly defined by the first order condition for equation (20)

$$
\begin{aligned}
\frac{\partial S^b}{\partial a} &= \int_{\beta}^{\zeta}\frac{\partial \Pi}{\partial a}dP \\
&= \int_{\beta}^{\zeta}\left(-\frac{\partial L}{\partial a} - 1\right)dP \\
&= 0
\end{aligned} \tag{22}
$$

Suppose the second order condition for a maximum holds. Then the function $a^b(i)$ exists and its derivative is

$$
\frac{da^b}{di} = -\frac{\dfrac{\partial^2 S^b}{\partial a\,\partial i}}{\dfrac{\partial^2 S^b}{\partial^2 a}} \le 0 \tag{23}
$$

if the numerator non-positive. Suppose that $\left(\partial I/\partial\omega - \partial L/\partial\omega\right) > 0$ so that the corporate loss distribution has more weight in its tails. Then observe that

$$\frac{\partial^2 S^b}{\partial a\,\partial i} = \frac{\partial}{\partial i}\left(\int_\beta^\zeta \left(-\frac{\partial L}{\partial a} - 1\right) dP\right)$$

$$= -\left(-\frac{\partial L}{\partial a} - 1\right) p(\beta)\frac{\partial \beta}{\partial i} \qquad (24)$$

$$< 0$$

The inequality follows because β is an increasing function of i and by the PDU the term in parentheses is positive at β.[10] The inequality in (24) yields a care function $a^b(i)$ that is decreasing in i.

Figure three and equation (20) are appropriate for some exercise prices but again there are two more generic cases that must also be included. First, there is an exercise price $\bar{i}$ sufficiently large that β equals δ; for larger i the boundary of the insolvency event remains at δ. For $i \geq \bar{i}$ the stock value is that provided in equation (2) and so $a^b(i)$ is a constant on the interval $i \geq \bar{i}$. Second, if there is an exercise price $i^\bullet$ such that β equals zero then $i^\bullet$ eliminates insolvency risk; for $i \leq i^\bullet$ the stock value is

$$S^b = \int_0^\gamma (\Pi + I - i)\,dP + \int_\gamma^\zeta \Pi\,dP$$

$$= \int_0^\gamma (\Gamma - a + (I - L) - i)\,dP + \int_\gamma^\zeta (\Gamma - L - a)\,dP \qquad (25)$$

and again it is apparent that $a^b(i)$ is a constant on the interval $i \leq i^\bullet$ if an $i^\bullet$ exists which makes β = 0. If such an $i^\bullet$ exists then below that threshold the care becomes the socially efficient choice as the following derivative shows

$$\frac{\partial S^b}{\partial a} = \int_0^\gamma \left(-\frac{\partial L}{\partial a} - 1\right) dP + \int_\gamma^\zeta \left(-\frac{\partial L}{\partial a} - 1\right) dP$$

$$= \int_0^\zeta \left(-\frac{\partial L}{\partial a} - 1\right) dP \qquad (26)$$

$$= 0$$

since (26) is equivalent to (8). The results are collected in figure four. Note that $a^b(i)$ is non-increasing and this is confirmation that the moral hazard problem can be resolved and provide the right incentives for an ILS issue.

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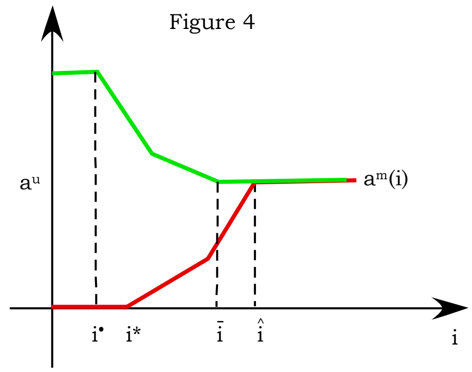
[10]Note that α is implicitly defined by the condition $\Pi(a,\beta) + (I(\beta) - i) = 0$ or equivalently by $\Gamma(\beta) - a - (L(a,\beta) - I(\beta)) - i = 0$ and so

$$\frac{\partial \beta}{\partial i} = \frac{1}{\frac{\partial I}{\partial \beta} - \frac{\partial L}{\partial \beta}} > 0\,.$$

Figure 4



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