A Technological Leadership Model

by

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Technological Leadership Notes

I. Introduction

In this essay, we consider the behavior of a firm which enters an otherwise competitive market possessing some monopoly control over its means of producing the homogeneous product sold in the market. The monopoly control over the new technology may be due to the proprietory nature of the information that the firm has concerning the nature of its new production process or it may be due to patent rights; in either case, we will suppose that the monopoly control of the new vintage technology lapses after T periods and the market once again becomes competitive. We are interested in the behavior of the new firm and its competitors in the short-run, or equivalently, in how monopoly control of the new vintage technology affects both the behavior of the firm which possesses it and the competitive fringe in the market.[1]

For simplicity, we will suppose that the demand for the product sold on this market is stable over time, i.e. if the demand at time t is characterized as $p_t = a_t - b_t\, q_t$, where q_t is the quantity demanded at date t, then it is the same for all dates t = 1, 2, ... Therefore, subsequently the demand at any date will simply be specified as p = a - b q. Next, suppose each firm in the industry has the same vintage "one" technology which is specified by its cost parameters m_1 and k_1; m_1 denotes the constant short-run average variable cost of production and k_1 is the unit cost of capital, i.e., both are dollar costs per unit of output. As more technologies are added in this industry suppose that they are indexed by j and represented by the cost parameters m_j and k_j. Finally, suppose that capital has an infinite life and that vintage j capital has a constant unit salvage value of s_j.[2]

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[1]Of course, to understand the behavior of the firms in the short-run we must also understand how the market adjusts in the long-run.

[2]The infinite life of capital assumption allows us to use a simple perpetuity formula to note that if k is the capital cost now for a unit of output then rk is the annualized cost, since

$$k = \frac{1}{r}\, r\, k.$$

The scrap value s_1 is simply the price of vintage one capital in a secondary market for capital equipment. This scrap value is the same in every time period, which in turn yields no economic depreciation.

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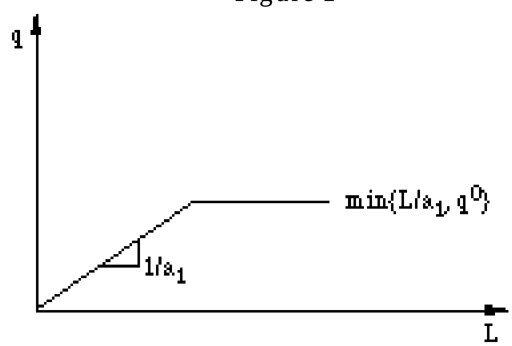
The assumption concerning costs is equivalent to assuming that firms have fixed proportions production functions of the form $q = \min\{L/a_{l1}, K/a_{k1}\}$ where L is the labor input, K is the capital input, a_{l1} is the labor input required per unit of output, and a_{k1} is the capital input required per unit of output.[3] In the short-run, labor is the only variable input; for a fixed capital input of K^0, the production function takes the form $q = \min\{L/a_{l1}, q^0\}$, where $q^0 = K^0/a_{k1}$ is the capacity output given the capital limitation K^0. The short-run version of the production function is shown in figure 1. Note that $1/a_{l1}$ is the slope of function for labor inputs between zero and $L^0 = a_{l1} K^0/a_{k1}$, and so it is the marginal productivity of labor; since the function is linear in that range it also follows that $1/a_{l1}$ is the average productivity of labor there. It follows that the short-run marginal and average variable cost are $m = w\, a_{l1}$, where w denotes the wage rate.[4]

Figure 1



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[3]The a_j's are input-output coefficients and are constant for this production function, which is also called the Leontief production function.

[4]Recall that

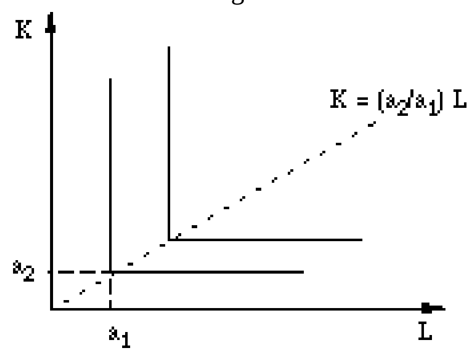
$$smc = \frac{w}{mp} \text{ and } savc = \frac{w}{ap}.$$

In this case, we have

$$smc = savc = \frac{w}{\frac{1}{a_{l1}}} = w\, a_{l1}.$$

In the long-run, capital and labor are both variable and the firm's production function is shown in figure 2. The firm determines its long-run

Figure 2



cost by selecting inputs to

$$\text{minimize } w\,L + r\,v\,K$$

$$\text{subject to } \min\left\{\frac{L}{a_{l1}}, \frac{K}{a_{k1}}\right\} = q,$$

where v is the price of capital, r is the interest rate, and q is a fixed quantity.[5] Since the firm selects its inputs so that $K = (a_{k1} / a_{l1})\,L$, for each fixed q, it follows that the minimum cost for the output q is $c = w\,L + r\,v\,(a_{k1} / a_{l1})\,L = [w\,a_{l1} + r\,v\,a_{k1}]\,(L / a_{l1}) = [w\,a_{l1} + r\,v\,a_{k1}]\,q$. Therefore, the long-run cost function is ltc(q) = lac q, where lac is a constant equal to $w\,a_{l1} + r\,v\,a_{k1} = m + r\,k_1$, i.e., we define k_1 to be $v\,a_{k1}$. Since long-run average cost is a constant, it follows that long-run marginal cost equals long-run average cost, i.e., $lmc_1 = lac_1 = m_1 + r\,k_1$.

[5]Note that v K is the total cost of the capital input but capital has an infinite life and so we are using the annualized cost r v K.

Having specified the demand and cost conditions in this market, we turn our attention to how the firms in the industry adjust their capital stock to achieve a long-run equilibrium position in a competitive market. This is done in section II, for the case in which all firms have the same vintage one technology. Then, in section III, we turn our attention to the case in which a new firm entering the market has monopoly control of a new vintage two technology for T periods, after which the monopoly control lapses.

II. The Competitive Model

Suppose all the firms in the industry have the same vintage one technology specified by its cost parameters m_1 and k_1. Let q denote the output of the industry. For a particular choice of q, the capital cost of the industry may be represented as k_1 q. For the same output choice, the industry generates a quasi-rent of $(p - m_1)\,q$ in each subsequent period; the present value of the quasi-rent cash flow is $(p - m_1)\,q/r$.[6] It follows that the net present value of the industry is

$$npv = -k_1 q + \frac{(p - m_1)\,q}{r} = \left[-k_1 + \frac{p - m_1}{r}\right] q.$$

Notice that as long as $(p - m_1)/r > k_1$ the industry and each firm in it is earning a positive net present value. This inequality simply says that the present value of the quasi-rent stream exceeds the capital cost required to generate that stream. As long as the inequality holds each firm in the industry has the incentive to increase its capital capacity and so its output and net

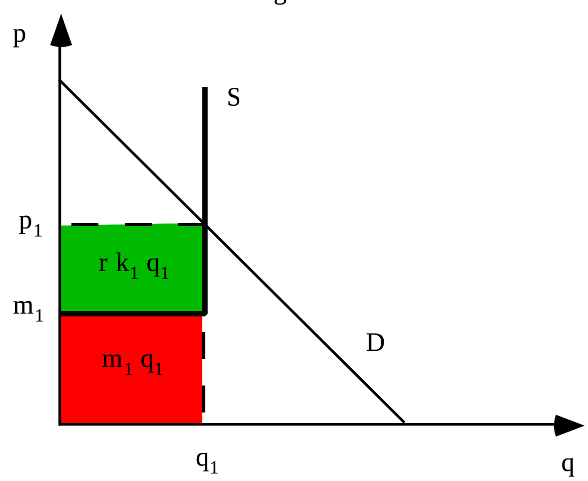
[6] The present value is pv where

$$pv = \sum_t \frac{(p_1 - m_1)\,q_1}{(1 + r)^t},$$

where the sum is over t = 1, 2, ... We simply observe that this sum converges to the limit given in the above text because the quasi-rent stream is a perpetuity.

present value. Under these circumstances the output of the industry increases and so the product price decreases. This process continues until any further profitable investment is eliminated, i.e., until $(p_1 - m_1)/r = k_1$. The industry capacity q_1 is the output level required to generate the price p_1, as shown in figure 3.[7] It may be noted that the industry price is $p_1 = m_1 + r\,k_1$, which is also the long-run marginal and average cost for this vintage technology. Therefore, market price equals long-run marginal cost equals long-run average cost, as is always the case in a competitive market.[8]

Figure 3



III. The Leadership Model

Now suppose that a new firm enters the market with a better vintage two technology, i.e., a better fixed proportions production process.[9] Suppose that the new firm maintains monopoly control of the vintage two technology for T periods, after which all firm may invest in

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[7]The supply function depicted is a short-run supply which is just the short-run marginal cost m_1 up to the capacity output of the industry.

[8]Equivalently, observe that the industry revenue is $p_1\,q_1 = m_1\,q_1 + r\,k_1\,q_1$, as shown in figure 3.

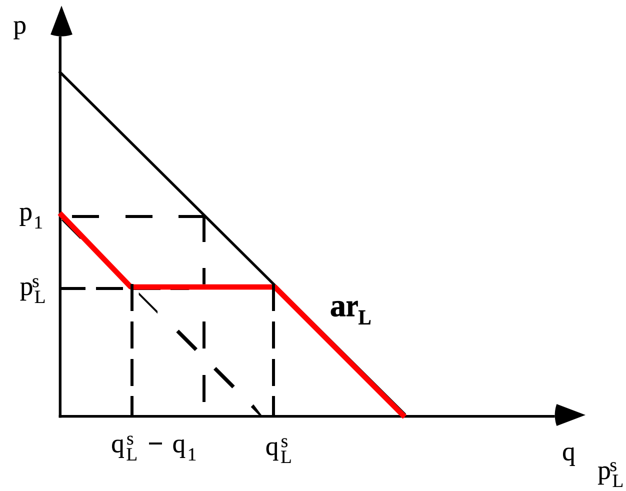
[9]The notion of better is simply that the long-run marginal cost of the vintage two technology is less than the long-run marginal cost of vintage one, i.e.,

$$m_2 + r\,k_2 < m_1 + r\,k_1.$$

the better technology. The firm knows that the market will become competitive after the T periods and so the market price will fall to the new long-run marginal cost of the new vintage two technology, i.e. to the price $p_2 = m_2 + r\ k_2$. However, during the period of monopoly control, the new firm has the ability to control the market price. Let p_L denote the price set by the technological leader. Exercising its market control, this firm will set a price in the interval (p_1, p_2). The leader sets its capacity output to maximize net present value, taking the reaction of its competitors into account.

Consider how the firms with the vintage one technology respond to the leader's output decisions. If the leader produces for this market then the leader must set a price p_L below p_1. The other firms in the market must determine whether to continue producing using vintage one capital or scrap some of it. The quasi-rents generated by vintage one are now $p_L - m_1$ per unit of output while the scrap value of vintage one capital is s_1 per unit of output. Hence, the rate of return on vintage one capital is $p_L - m_1 / s_1$. If this rate of return exceeds the rate of return in the economy's financial markets then firms with vintage one will not scrap, i.e. $p_L - m_1 / s_1 > r$. If $p_L - m_1 / s_1 < r$ then firms with vintage one will scrap. At a price $p_L^s = m_1 + r\ s_1$, firms with vintage one are indifferent between scrapping and not scrapping because the rate of return on vintage one capital is equal to the rate of return in the financial markets. With this knowledge of other firms' scrapping decisions, the leader firm can determine its residual demand, as shown in figure 4. The residual demand $p_L = ar_L$ is $p_L = a - bq_1 - bq_L$, if q_L $(0, q_L^s - q_1)$, $p_L = p_L^s$, if q_L $(q_L^s - q_1, q_L^s)$, and $p_L = a - bq_L$, if q_L $(q_L^s, \frac{a}{b})$.

Figure 4



Using the residual demand to determine the prices that can be set during the period of monopoly control, the leader determines the capacity of the vintage two technology which maximizes net present value. The net present value of a capacity q_L is

$$npv_L = -k_2\, q_L + \sum_{t=1}^{t=T} \frac{(p_L - m_2)q_L}{(1+r)^t} + \sum_{t=T+1}^{t=\infty} \frac{(p_2 - m_2)q_L}{(1+r)^t}$$

This net present value may be simplified by noting that the quasi-rents during the monopoly control period are constant and so the cashflow is an annuity and that the quasi-rents during the competitive periods is a perpetuity from the perspective of year T.[10] Therefore, letting d_T denote the annuity discount factor for T periods, note that npv_L may be rewritten as

$$npv_L = -k_2\, q_L + [(p_L - m_2)\, q_L]\, d_T + \frac{(p_2 - m_2)\, q_L}{r\ (1+r)^T}$$

10 Note that

$$\sum_{t=T+1}^{t=\infty} \frac{1}{(1+r)^t} = \frac{1}{r\,(1+r)^T}$$

$$= q_L\{-k_2 + (p_L - m_2)\ d_T + (p_2 - m_2)\ (-d_T + \frac{1}{r})\} =$$

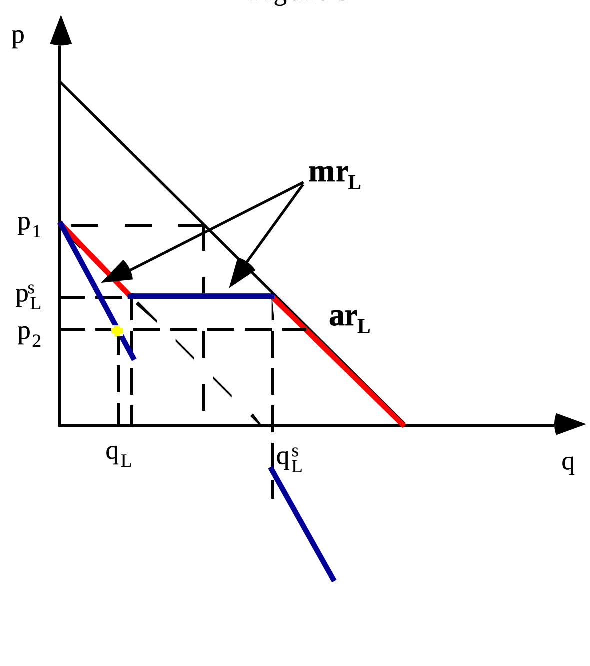
$$= q_L\{-\ k_2 + \frac{p_2 - m_2}{r} + [(p_L - m_2) - (p_2 - m_2)]\ d_T\}$$

$$= (p_L -\ lmc_2)\ q_L\ d_T.$$

Thus the net present value of the leader is the present value of the monopoly rents, i.e. the quasi-rent in excess of the capital costs of the vintage.

In maximizing net present value, the leader must determine the marginal revenue and marginal cost. Since the leader's demand has two kinks, i.e., at q_L^s - q_1 and q_L^s, the marginal revenue function has two points of discontinuity, as shown in figure 5. Since p_2 is the marginal cost of the leader's vintage two technology, the leader maximizes net present value where marginal revenue equals marginal cost or where marginal revenue exceeds marginal cost for the last unit produced but not the next unit. These output levels are represented by q_L^* and q_L^s in figure 5.

Figure 5



If the leader selects the output q_L* then the shaded area in figure 6 represents the monopoly rent generated per period. Alternatively, if the leader selects q_L^s then the shaded area in figure 7 represents the monopoly rent generated period. Of course, the leader selects the output level which generates the largest monopoly rent per period.

Figure 6

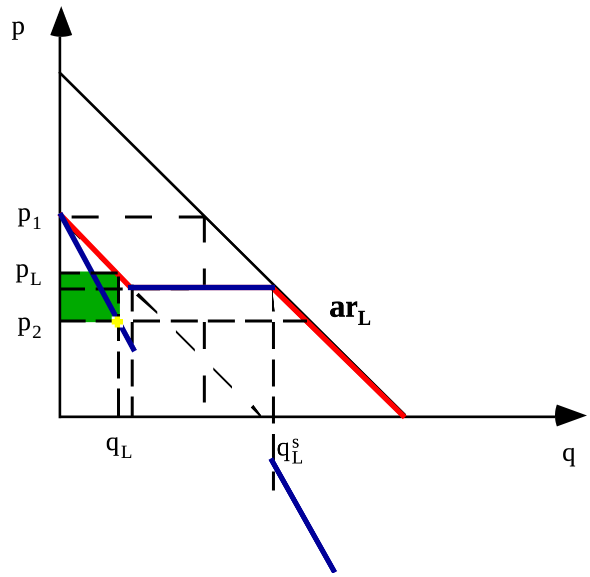


Figure 7